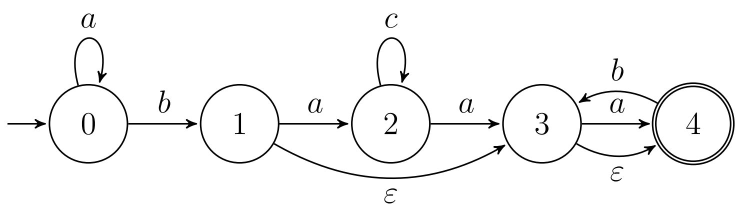
Langages Formels - TD 3

February 16, 2024

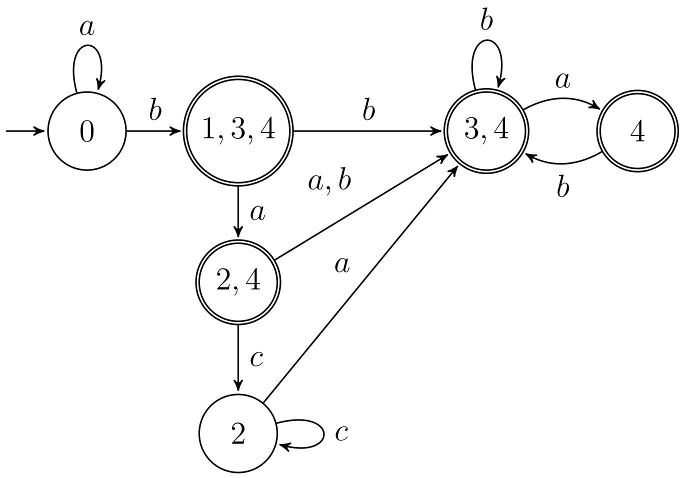
Exercise 1 : Déterminisation

Quel est le langage reconnu par $\mathcal{A}$? Déterminisez $\mathcal{A}$.



(a) Automate $\mathcal{A}$

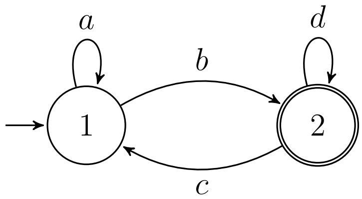
Cet automate reconnait $a^*b(ac^*a+\varepsilon)(b+ab)^*(a+\varepsilon)$.



(b) Automate $\mathcal{A}$ déterminisé

Exercise 2 :

Apply the algorithm of McNaughton-Yamada on the following automaton. Detail each step.



Cet automate reconnait $a^*b(d + ca^*b)^*$.

Exercise 3 : Brzozowski-McCluskey algorithm

The goal of this exercise is to translate a finite automaton into a rational expression, giving an alternate proof of the associated implication of Kleene's theorem. We will proceed by successive transformations of the automaton.

1. We call *strongly normalized* every automaton which has a unique initial state to which no transition leads and a unique final state with no exiting transition, i.e. an automaton $\mathcal{A} = \langle Q, \Sigma, \{i\}, \{f\}, \delta\rangle$ such that for every state q and letter a, $(q, a, i) \notin \delta$ and $(f, a, q) \notin \delta$. Show that for all finite automaton, there is a strongly normalized automaton which recognizes the same language.

$Q \leftarrow Q \cup \{i, f\}$, $I \leftarrow \{i\}$, $F \leftarrow \{f\}$,
$\delta \leftarrow \delta \cup \{(i, a, q) | (q_i, a, q) \in \delta, q_i \in I\} \cup \{(q, a, f) | (q, a, q_f) \in \delta, q_f \in F\}$. Si il y a un état commun à I et F, on rajoute une ε-transition de i à f.

We will use a generalization of the definition of finite automata: the transition function will be a subset of $Q \times 2^{\Sigma^*} \times Q$. An execution of such an automaton recognizes the concatenation of languages of the transitions' labels. The automaton recognizes the union of the languages of all its accepting executions.

2. Show that every generalized automaton is equivalent to a generalized automaton in which there exists exactly one transition between each pair of states: $q' \in \delta(q, L)$ et $q' \in \delta(q, L')$ implies $L = L'$.

On peut remplacer les transitions $q \xrightarrow{L} q'$ et $q \xrightarrow{L'} q'$ par une unique transition $q \xrightarrow{L+L'} q'$. S'il n'y a pas de transition $q \xrightarrow{L} q'$, on ajoute $q \xrightarrow{\emptyset} q'$.

3. Let $\mathcal{A}$ be a strongly normalized generalized automaton with initial state i and final state f. Let $q \in Q_{\mathcal{A}}$, $q \notin \{i, f\}$. Show that there exists an automaton equivalent to $\mathcal{A}$ with set of states $Q_{\mathcal{A}} \setminus \{q\}$.

Pour tout $q_1, q_2 \neq q$, on peut remplacer les transitions $q_1 \xrightarrow{L_1} q$, $q \xrightarrow{L_3} q_2$, $q \xrightarrow{L_2} q$ et $q_1 \xrightarrow{K} q_2$ par une transition $q_1 \xrightarrow{L_1 \cdot L_2^* \cdot L_3 + K} q_2$.

4. Conclude that if L is recognized by a strongly normalized generalized automaton $\mathcal{A}$, then L belongs to the rational closure of the labels of the transitions of $\mathcal{A}$.

Si L est reconnu par un snga $\mathcal{A}$, alors il est reconnu par un snga à deux états $i \xrightarrow{L} f$ construit en suivant les deux questions précédentes. Donc L appartient à la fermeture par somme et concaténation et étoile des étiquettes de $\mathcal{A}$.

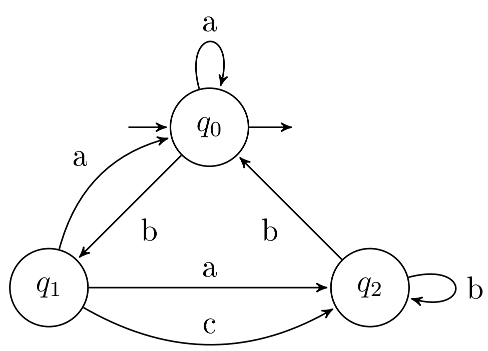
5. Show that every finite automaton has an equivalent generalized automaton.

On remplace chaque étiquette a par le langage $\{a\}$.

6. Give a procedure which, given a finite automaton, outputs a rational expression of same language.

On applique les questions 1,5,2,3.

7. Apply the construction to compute a rational expression corresponding to the following automaton:



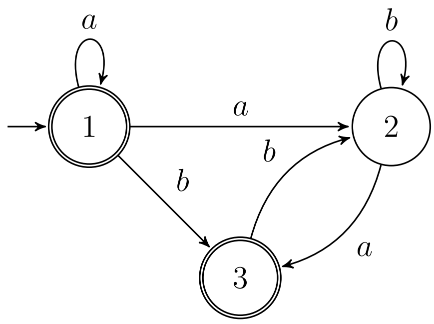
$(a + b(a + (a + c)b^+)^*$

Contrôle continu 3

À rendre pour le 15/02 à 16h15.

Exercise 4 :

Apply the algorithm of McNaughton-Yamada on the following automaton. Detail each step.



$e = a^* + (a^*b + a^+b^*a)(b^+a)^*$

Exercise 5 : A rational half ?

Let L be a rational language over a finite alphabet Σ. Show that $\mathsf{Half}(L) = \{ f \in \Sigma^* \ : \ ff \in L \}$ is rational.

Yes, you can build the automaton : $Q' = Q \times Q \times Q$, $I' = \{(i,q,q) | q \in Q, i \in I\}$, $F' = \{(q,q,f) | q \in Q, f \in F\}$, and $\delta'_a(p_1, q, p_2) = \{ \ (q_1, q, q_2) \ : \ \delta_a(p_1) = q_1, \delta_a(p_2) = q_2 \ \}$.

Is $\mathsf{FH}(L) = \{ \ f \in \Sigma^* \ : \ \exists h \in \Sigma^*. \ |h| = |f|, \ f \in L \ \}$ rational?

Same idea : $Q' = Q \times Q \times Q$, $I' = \{(i,q,q) | q \in Q, i \in I\}$, $F' = \{(q,q,f) | q \in Q, f \in F\}$, and $\delta'_a(p_1, q, p_2) = \{ \ (q_1, q, q_2) \ : \ \delta_a(p_1) = q_1, \delta_b(p_2) = q_2$ for any $b \in \Sigma \ \}$.

Is $\mathsf{Double\text{-}pad}(L) = \{ \ fh \in \Sigma^* \ : \ \ f \in L, \ h \in \Sigma^*, \ |h| = |f| \ \}$ rational?

No. Take for instance $L = 0^*$. If $\mathsf{Double\text{-}pad}(L)$ was rational, then $\mathsf{Double\text{-}pad}(L) \cap 0^*1^* = \{ \ 0^n1^n \ : \ n \in \mathbb{N} \ \}$ would be rational.