Langages Formels

TD 5

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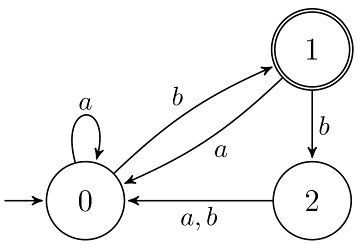
Exercise 1 : Recognizability by monoid

1. Give a finite monoid M, a morphism φ and a subset P of M which recognize the language accepted by the following automaton:



2. Prove that if L is regular, the language of its kth roots $\mathfrak{R}_k(L) = \{ u : u^k \in L \}$ is also regular for $k > 1$.
3. Build a ND automaton that recognizes $\mathfrak{R}_2(L)$ from the automaton that recognizes L.
4. Adapt this automaton such that it recognizes $FH(L) = \{ u : \exists v, |v| = |u|, uv \in L \}$.

Exercise 2 : Congruences and monoids

An equivalence relation R on Σ^* is a *congruence* if uRv implies $xuyRxvy$ for all x, y. We will call *congruence classes* the equivalence classes of a congruence.

1. Prove that a language is regular iff it is the union of some of the congruence classes of a congruence relation of *finite index*, i.e. with a finite number of congruence classes.

A congruence c_1 is *coarser* (i.e. "grossière") than another congruence c_2 if every congruences classe of c_2 is included in a congruence class of c_1.

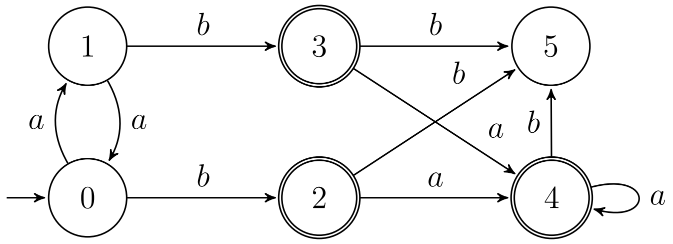
2. Let L be a language. Find a characterization of the coarsest congruence $\equiv_L$ such that L is the union of some of its congruence classes.

This congruence is called the *syntactic* congruence of L.

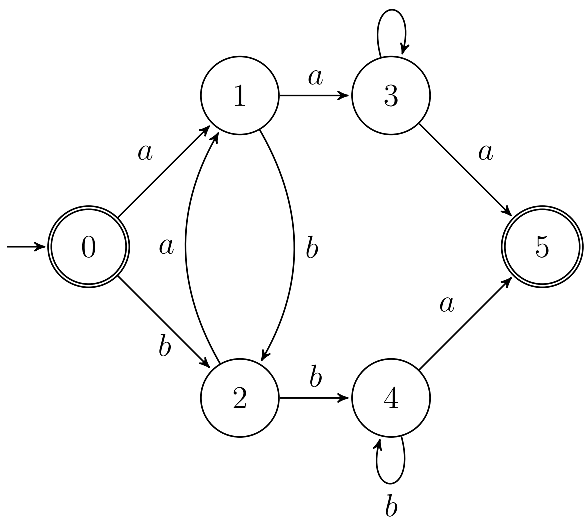
3. Quel propriété de la congruence syntaxique de $\mathcal{L}$ est vraie ssi $\mathcal{L}$ est reconnaissable ? Utilisez-ça pour montrer que $\{ a^n b^n \ : \ n \in \mathbb{N} \}$ n'est pas reconnaissable.
4. We know that a language of Σ^* is regular iff there exists a finite monoid $(M, \times)$, a morphism $\mu : (\Sigma^*, \cdot) \to (M, \times)$, and a set $P \subseteq M$ such that $L = \mu^{-1}(P)$. Find a characterization of the smallest such monoid for a regular language L.
5. What is the link between the syntactic congruence, this smallest monoid, and the minimal automaton?

Exercise 3 : Minimisation

Miniminisez les automates suivants. Quels langages reconnaissent-ils ?



(a) Automate $\mathcal{A}_1$



(b) Automate $\mathcal{A}_2$

Contrôle continu 6

À rendre pour le 05/03 à 16h15.

Rappel : Votre note finale de contrôle continu sera la moyenne de vos trois meilleurs notes. Vous pouvez me rendre autant de contrôles continus que vous voulez, mais je vous recommande de m'en rendre au moins 3. Celui-ci est le dernier (après ça recommence à zéro pour la seconde partie du cours).

Exercise 4 : Aperiodic languages

A language is *aperiodic* when its syntactic monoid $(M, \cdot)$ is aperiodic, i.e. for all $x \in M$ there exists $n \in \mathbb{N}$ such that $x^n = x^{n+1}$.

1. A finite deterministic complete automaton has a *counter* when there exists $n > 1$, a sequence of distinct states $q_0, \dots, q_{n-1}$ and a word $w \in \Sigma^*$ such that $\delta(q_i, w) = q_{i+1 \mod n}$ for all $i \in \{ 0, \dots, n-1 \}$.
   Show that $L \subseteq \Sigma^*$ is aperiodic iff its minimal automaton has no counter.
2. Show that if a language is *star-free*, i.e. in the smallest class containing the letters of the alphabet and closed by union, concatenation, and complement, then it is aperiodic. [1]
3. For the following languages, show if it is aperiodic or not:
   (a) $(ab)^*$,
   (b) $(aa)^*$,
   (c) $(a(ab)^*b)^*$,
   (d) $(ab + ba)^*$.

[1] The converse also holds, but is much harder to prove.