



Laboratoire
Méthodes
Formelles

université
PARIS-SACLAY



école
normale
supérieure
paris—saclay

On the Probabilistic and Statistical Verification of Infinite Markov Chains

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France

Joint work with Benoît Barbot (LACL) and Serge Haddad (LMF)

Work partly supported by ANR projects MAVeriQ and BisoUS

General purpose

Design algorithms to estimate probabilities in some **infinite-state** Markov chains, **with guarantees**

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Design algorithms to estimate probabilities in some **infinite-state** Markov chains, **with guarantees**

Our contributions

- ▶ Review two existing approaches (approximation algorithm and estimation algorithm) and specify the required hypothesis for correctness
- ▶ Propose an approach based on **importance sampling** and **abstraction** to partly relax the hypothesis
- ▶ Analyze empirically the approaches

Discrete-time Markov chains

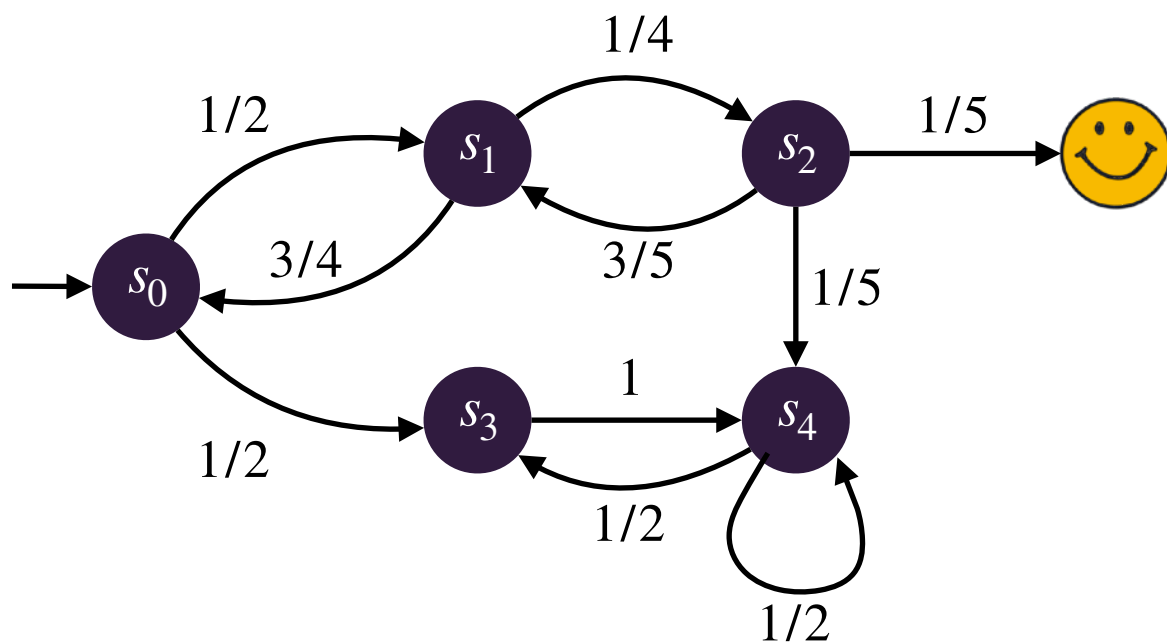
Discrete-time Markov chain (DTMC)

$\mathcal{C} = (S, s_0, \delta)$ with S at most denumerable, $s_0 \in S$ and $\delta : S \rightarrow \text{Dist}(S)$

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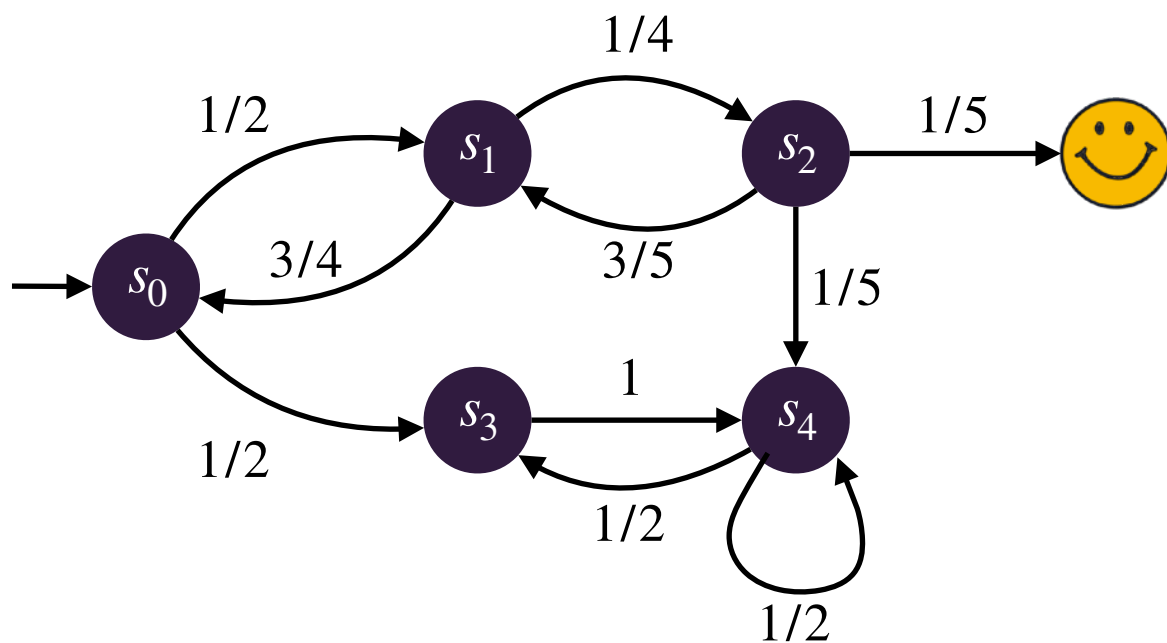


Finite Markov chain

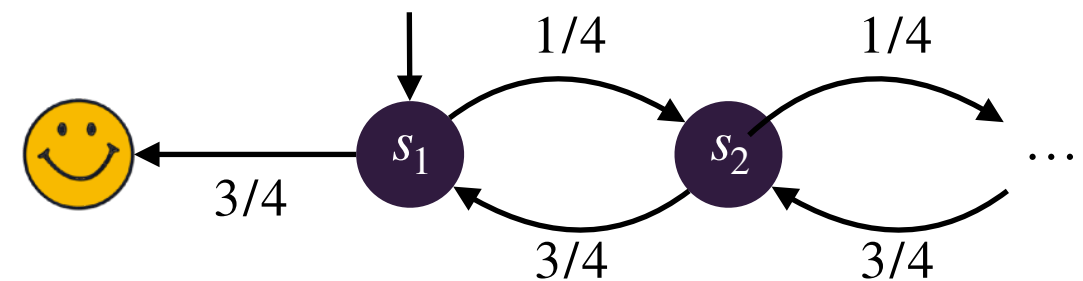
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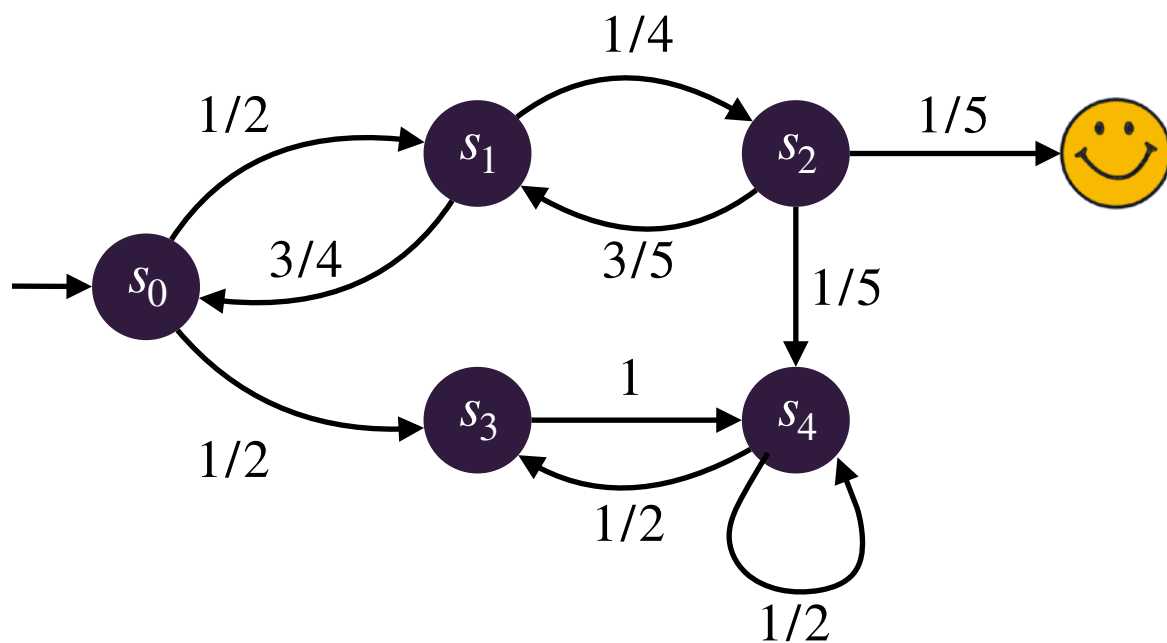
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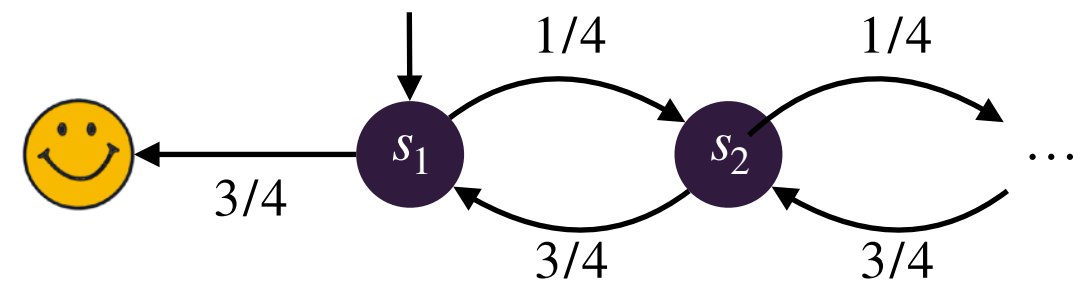
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+ effectivity conditions...



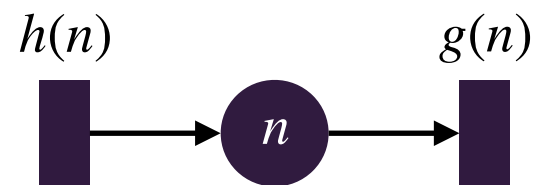
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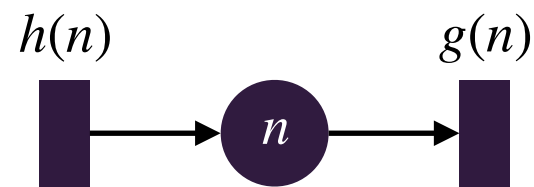
High-level models for (infinite) Markov chains

- ▶ Queues



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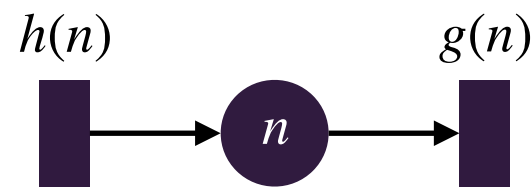
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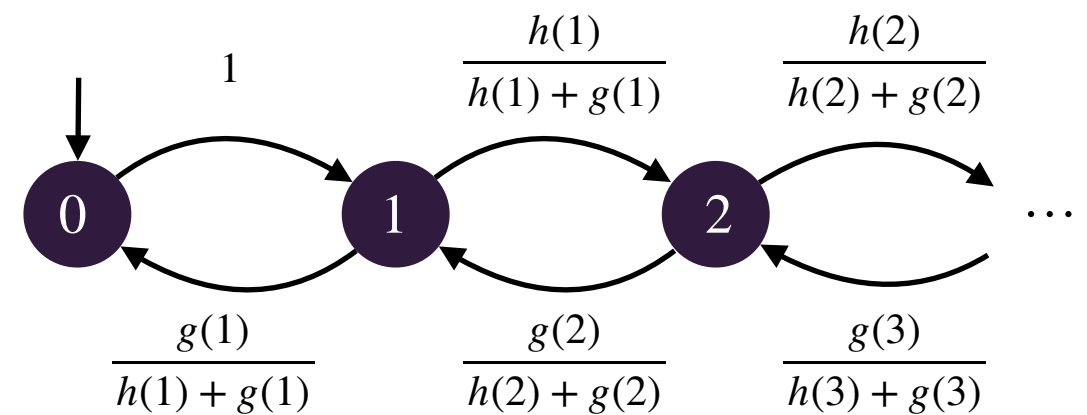
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High-level models for (infinite) Markov chains

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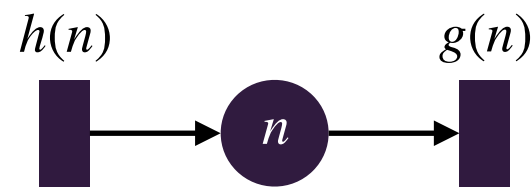


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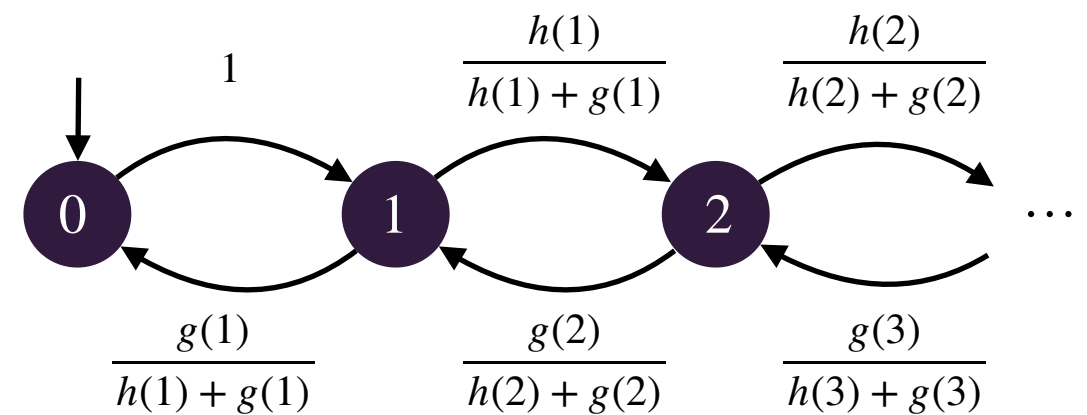


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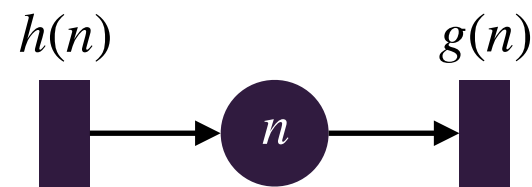


► Probabilistic pushdown automata

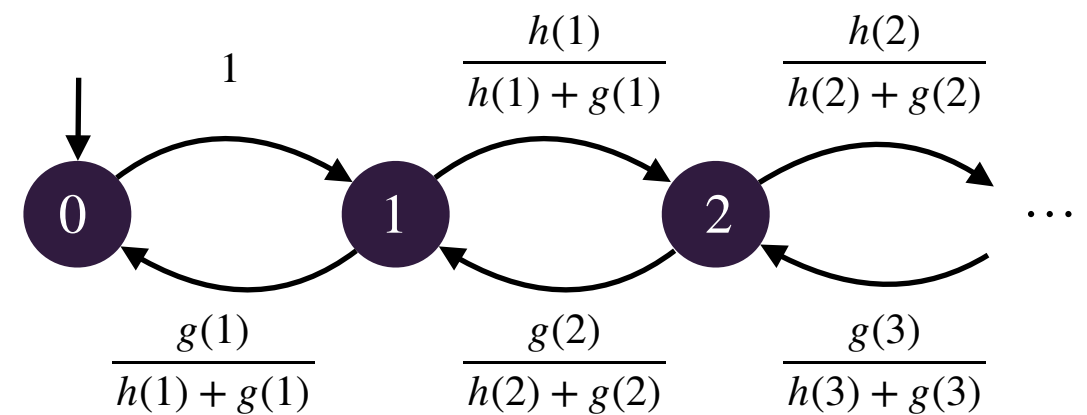
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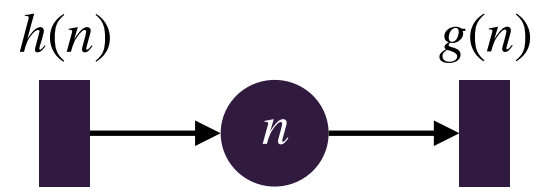
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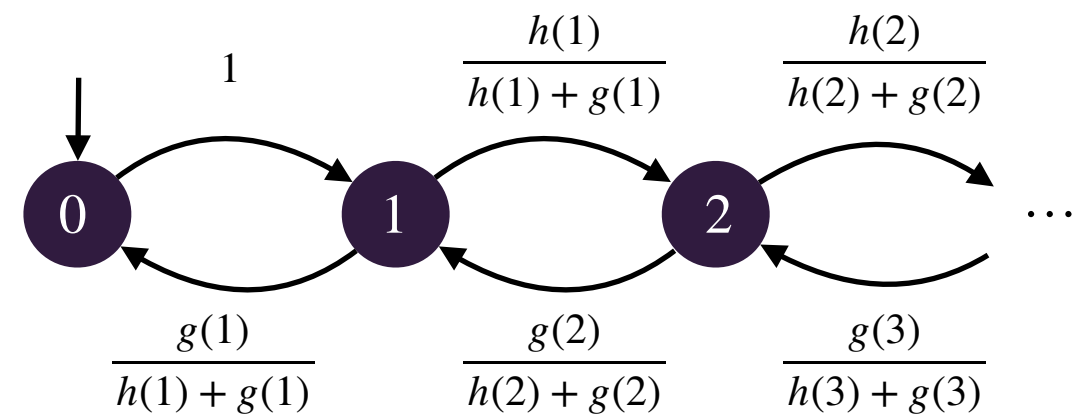
n is the height of the stack

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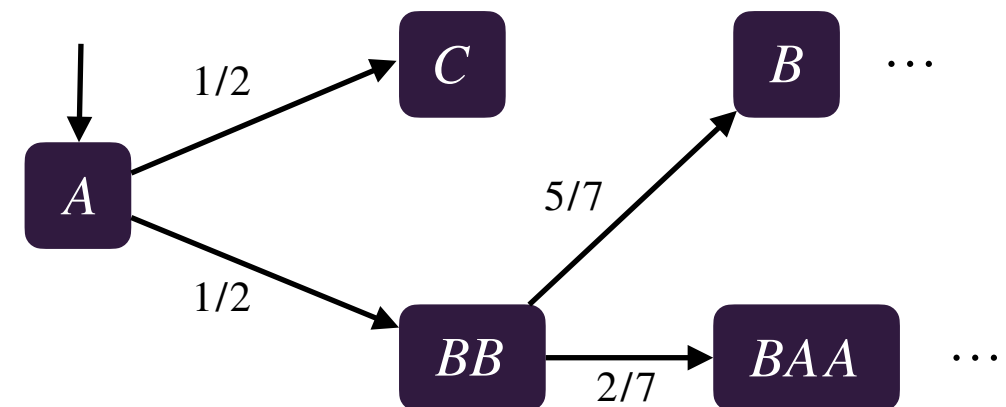
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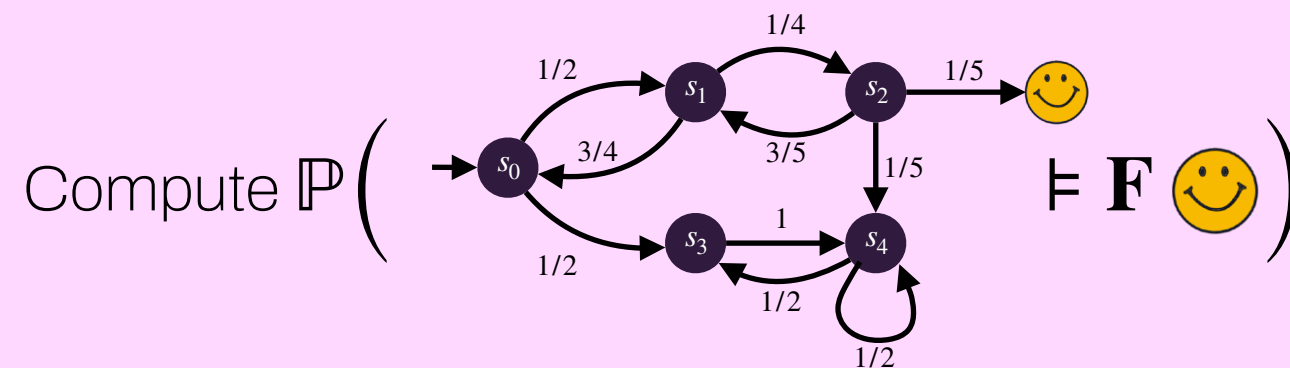
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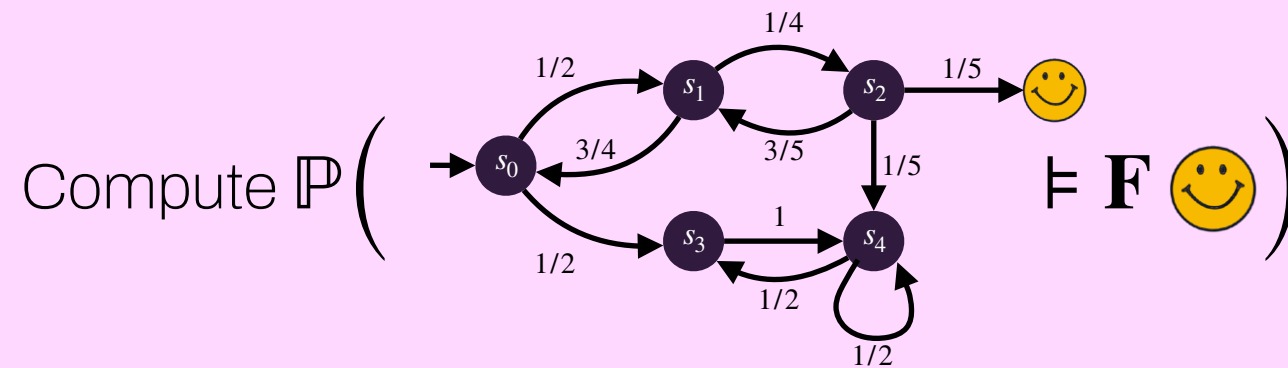
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Quantitative analysis of Markov chains



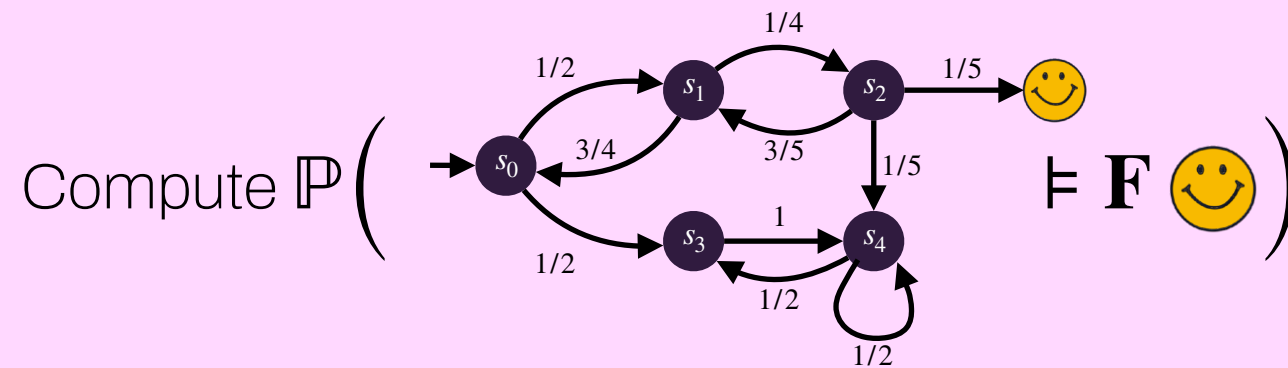
Quantitative analysis of Markov chains



Closed-form solution

- ▶ Random walk of parameter $p > 1/2$:
 $\mathbb{P}_{s_n}(\mathbf{F} \text{ } \text{😊}) = \kappa^n$, where $\kappa = \frac{1-p}{p}$
- ▶ Does not always exist

Quantitative analysis of Markov chains



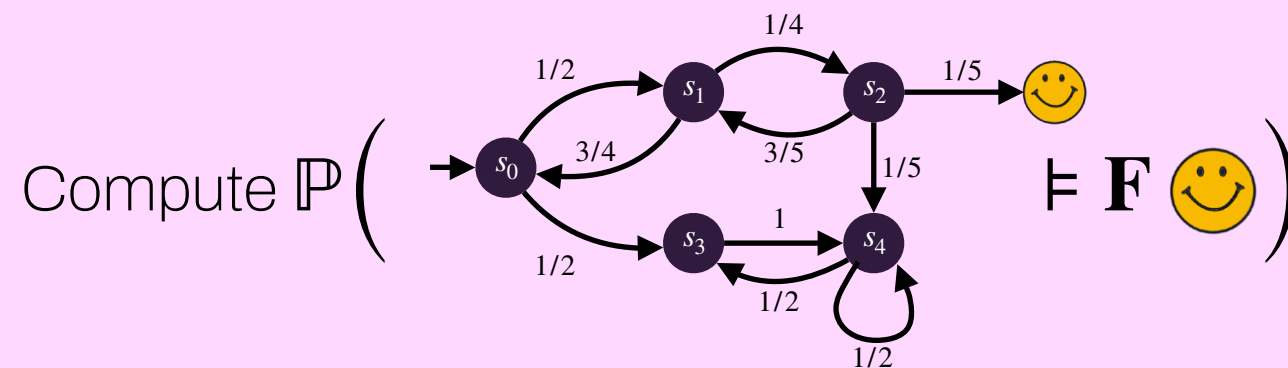
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$$x_s = \begin{cases} 1 & \text{if } s = \text{😊} \\ 0 & \text{if } s \not\models \exists \mathbf{F} \text{ } \text{😊} \\ \sum_t \mathbb{P}(s \rightarrow t) \cdot x_t & \text{otherwise} \end{cases}$$
- ▶ $\mathbb{P}_{s_0}(\mathbf{F} \text{ } \text{😊}) = 1/19$
- ▶ System must be finite
- ▶ Prone to numerical error

Quantitative analysis of Markov chains



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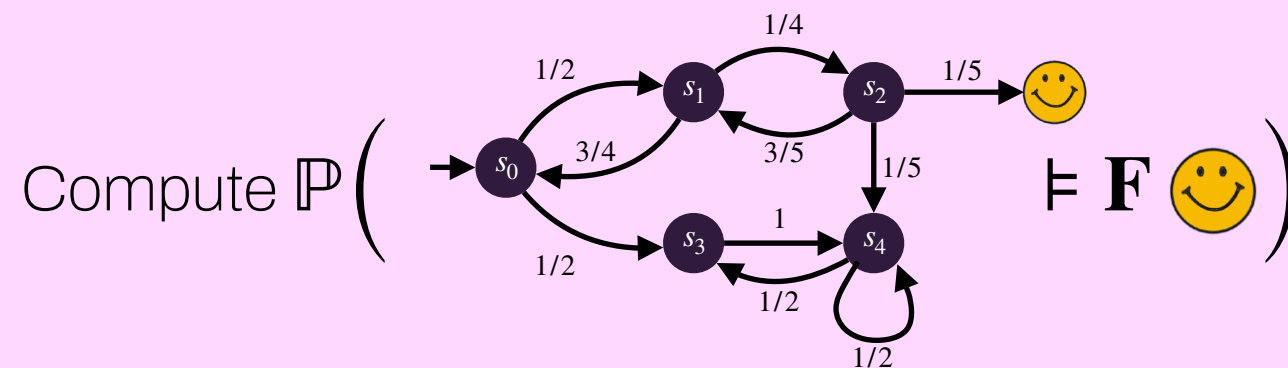
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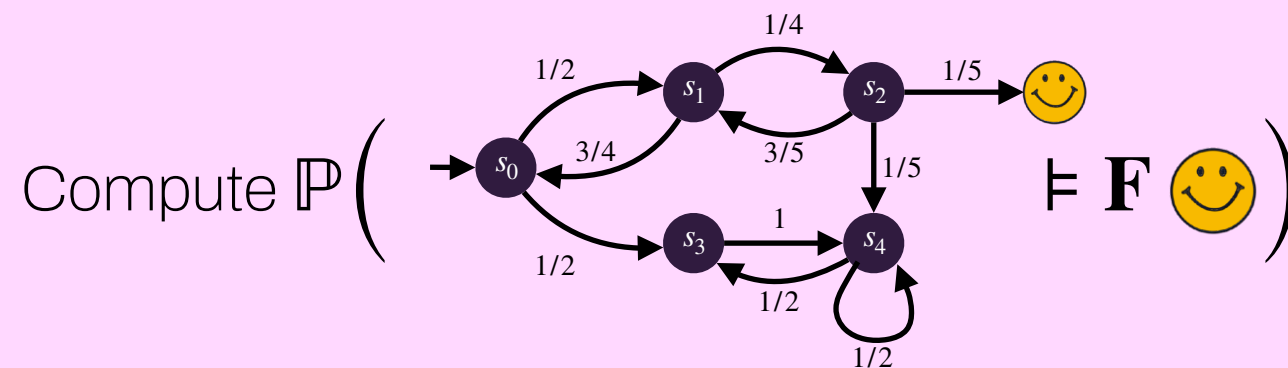
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- ▶ Ad-hoc methods in specific classes

Quantitative analysis of Markov chains



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- ▶ No general method exists for infinite Markov chains
- ▶ Ad-hoc methods in specific classes
- ▶ Specific approaches for **decisive** Markov chains

Decisiveness

$$\text{☹️} = \{s \in S \mid s \not\models \exists \mathbf{F} \text{☺️}\}$$

Decisiveness

A DTMC $\mathcal{C}$ is **decisive** from s w.r.t. ☺️ if $\mathbb{P}_s(\mathbf{F} \text{☺️} \vee \mathbf{F} \text{☹️}) = 1$

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- ▶ Examples of decisive Markov chains: finite Markov chains, probabilistic lossy channel systems, probabilistic VASS, noisy Turing machines, ...

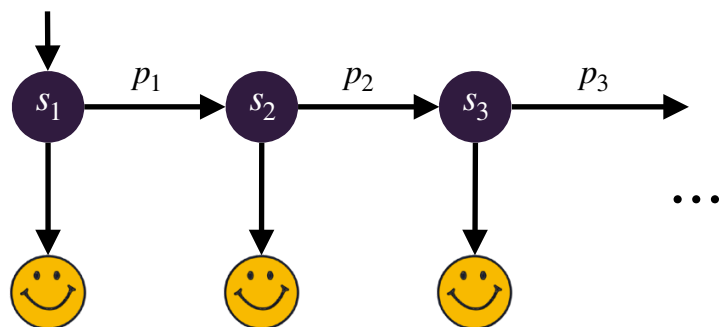
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- ▶ Examples of decisive Markov chains: finite Markov chains, probabilistic lossy channel systems, probabilistic VASS, noisy Turing machines, ...
- ▶ Example/counterexample:



$$\bullet \quad \mathbb{P}(\mathbf{G} \neg \text{☺️}) = \prod_{i \geq 1} p_i$$

- Decisive iff this product equals 0

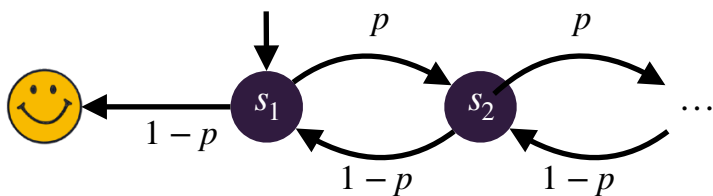
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- ▶ Examples of decisive Markov chains: finite Markov chains, probabilistic lossy channel systems, probabilistic VASS, noisy Turing machines, ...
- ▶ Example/counterexample:



- Recurrent random walk ($p \leq 1/2$): decisive
- Transient random walk ($p > 1/2$): not decisive

Deciding decisiveness?

Classes where decisiveness can be decided

- ▶ Probabilistic pushdown automata with constant weights [ABM07]
- ▶ Random walks with polynomial weights [FHY23]
- ▶ So-called probabilistic homogeneous one-counter machines with polynomial weights (this extends the model of quasi-birth death processes) [FHY23]

Approximation scheme

- ▶ Aim: compute probability of **F** 😊
- ▶ 😞 = $\{s \in S \mid s \not\models \exists \mathbf{F} \text{ 😊}\}$

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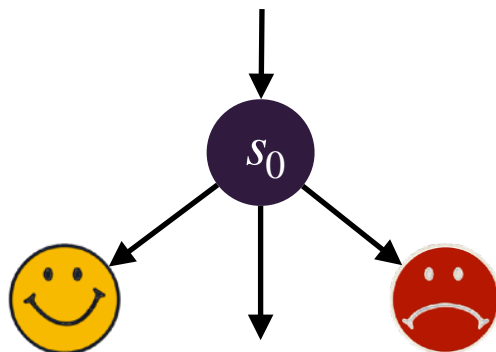
Given $\varepsilon > 0$, for every n , compute:

$$\begin{cases} p_n^{\text{yes}} &= \mathbb{P}(\mathbf{F}_{\leq n} \text{ 😊}) \\ p_n^{\text{no}} &= \mathbb{P}(\mathbf{F}_{\leq n} \text{ 😞}) \end{cases}$$

until $p_n^{\text{yes}} + p_n^{\text{no}} \geq 1 - \varepsilon$

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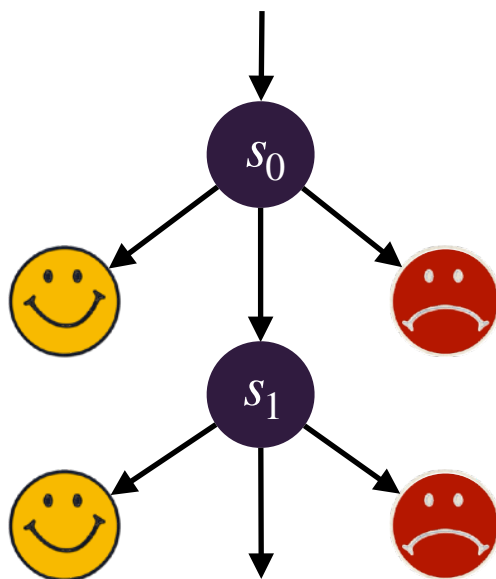
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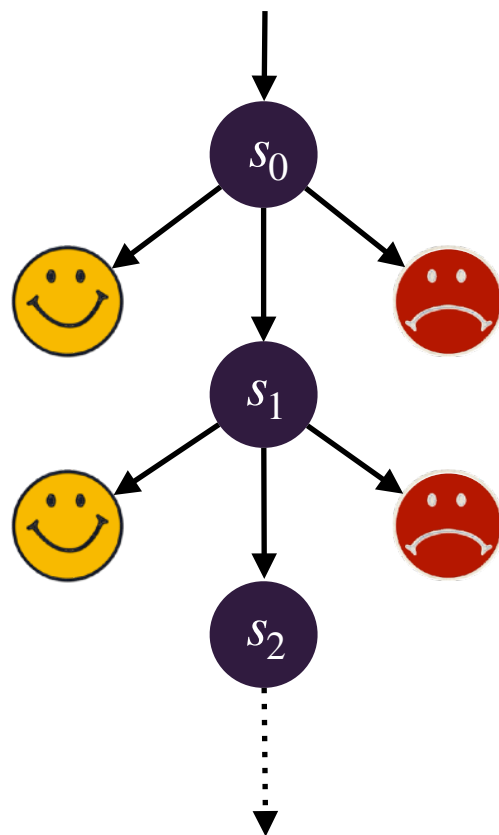
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$$\text{I} \wedge \qquad \qquad \qquad \text{V} \text{I}$$

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Approximation scheme

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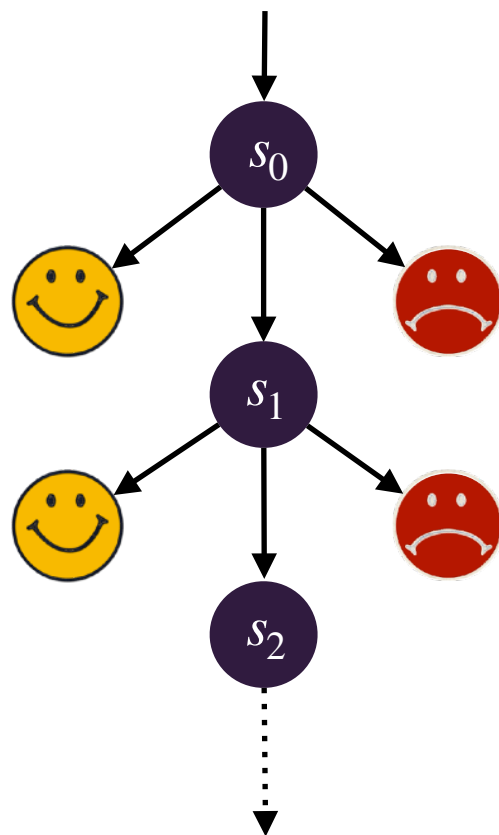
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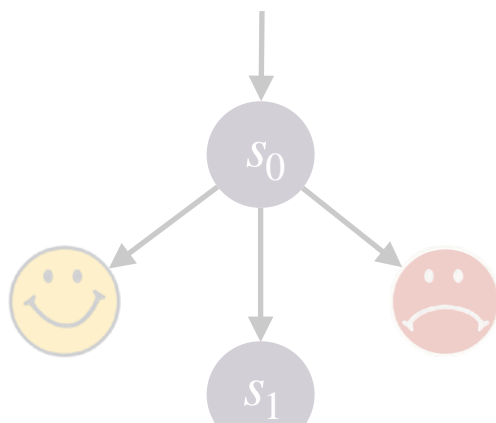
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At the limit: $\mathbb{P}(\mathbf{F} \text{ 😊}) \qquad \qquad \qquad 1 - \mathbb{P}(\mathbf{F} \text{ 😞})$

Approximation scheme

- ▶ Aim: compute probability of $\mathbf{F}$ 😊
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The approximation scheme converges
iff
 $\mathcal{C}$ is decisive from s_0 w.r.t. 😊

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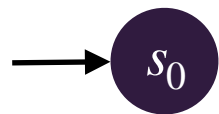
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$$\vdots$$

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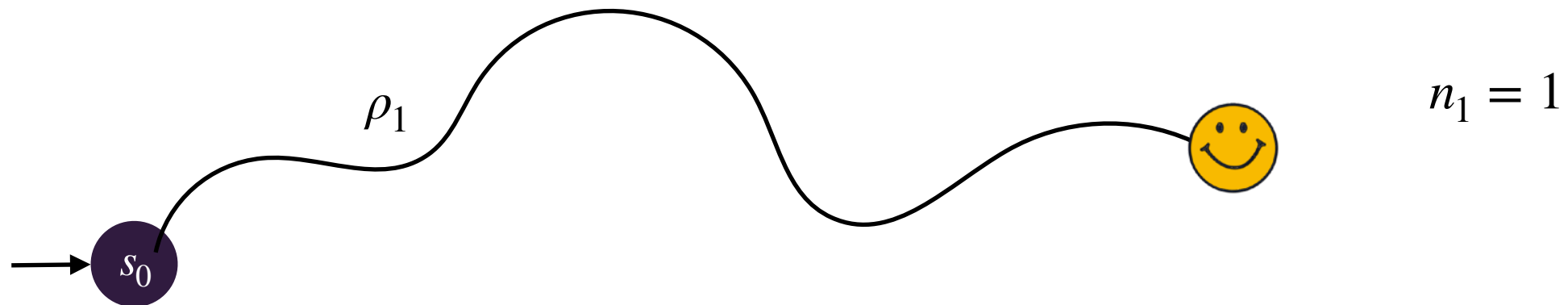
Statistical model-checking

Sample N paths



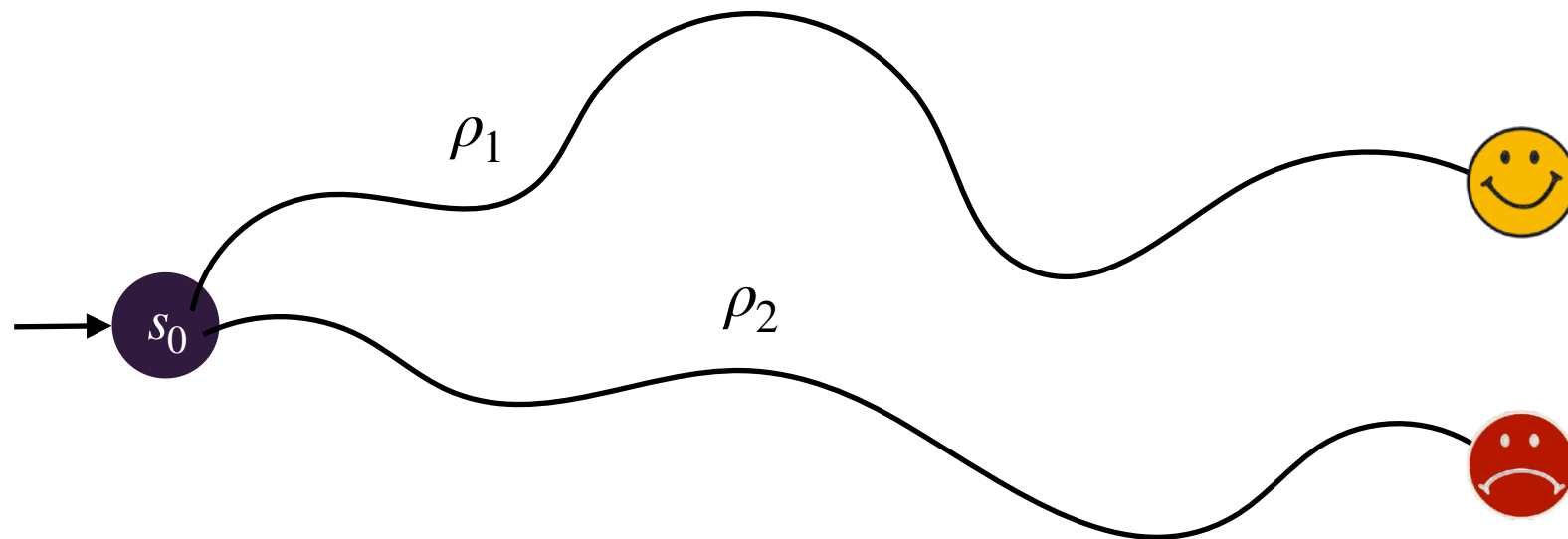
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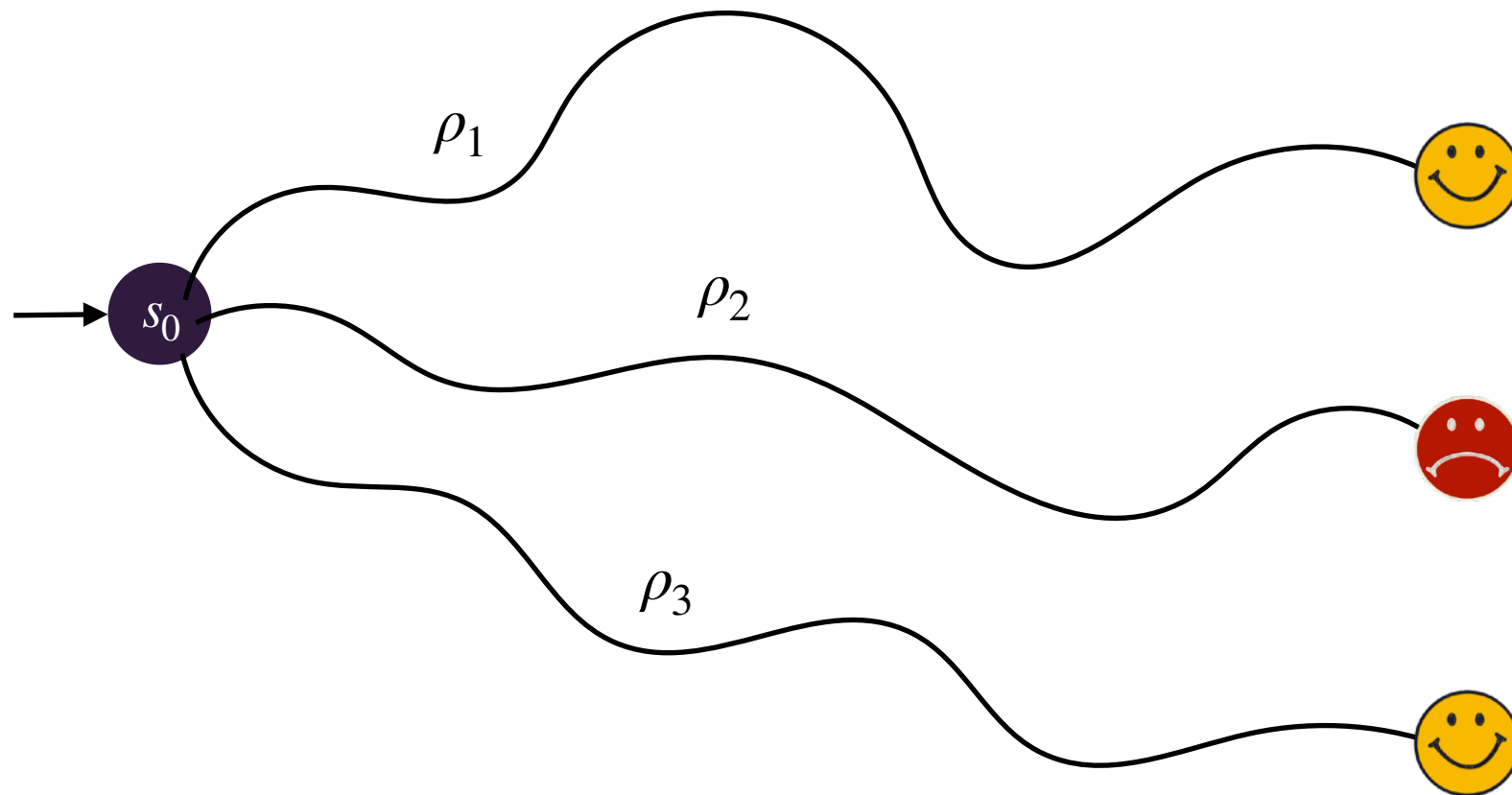


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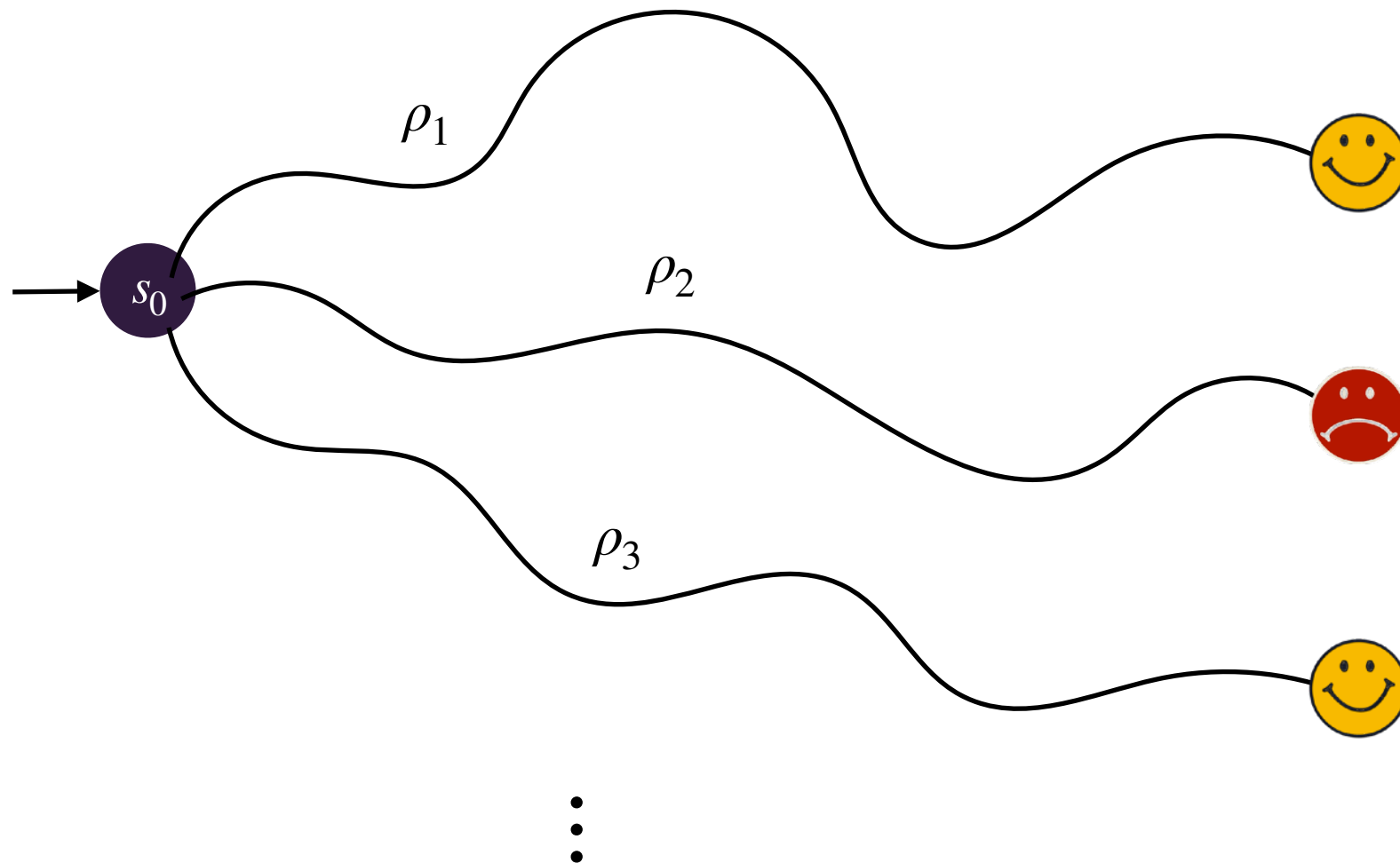
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Statistical model-checking

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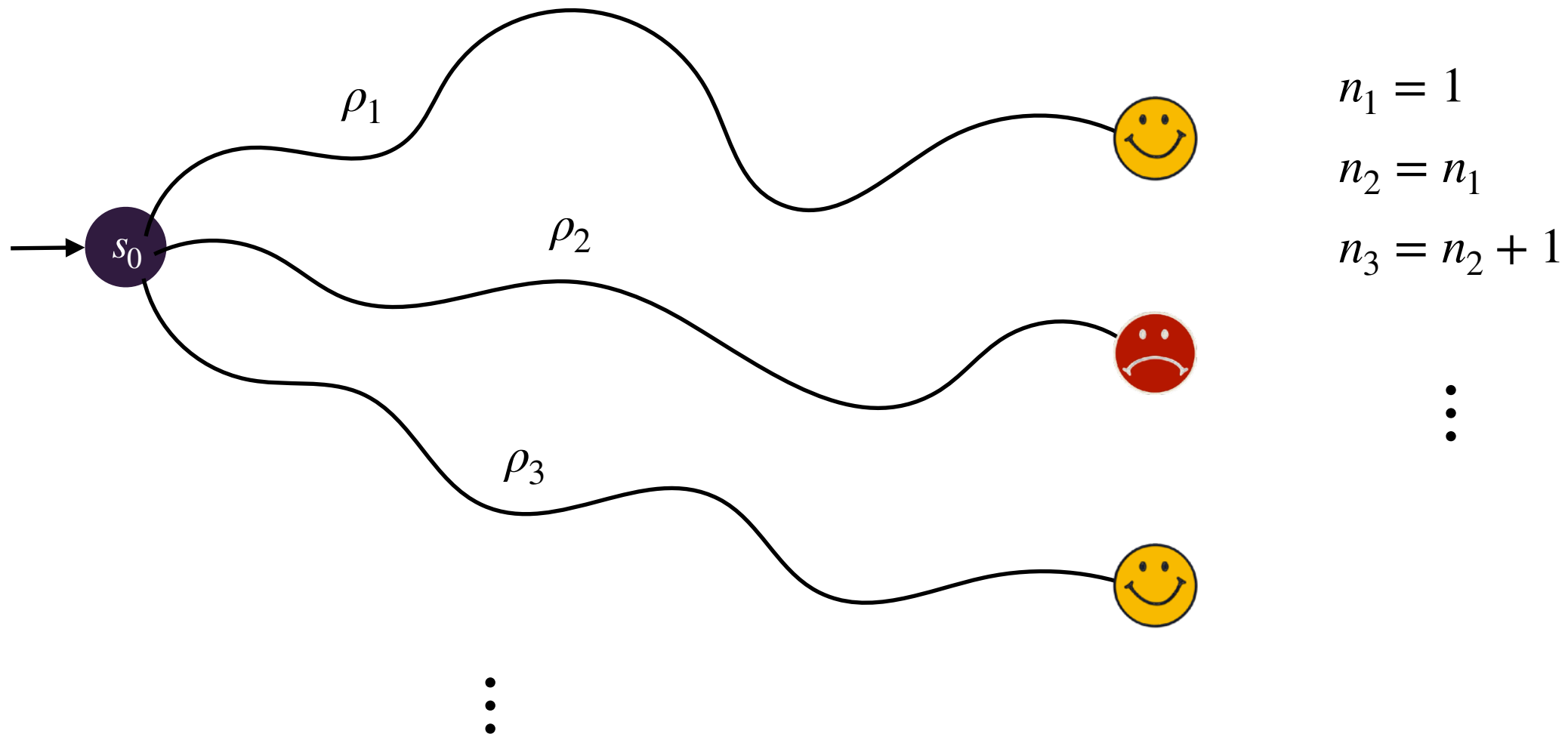
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⋮

Statistical model-checking

Sample N paths



Return $\frac{n_N}{N}$ + some confidence interval
(in the best case)

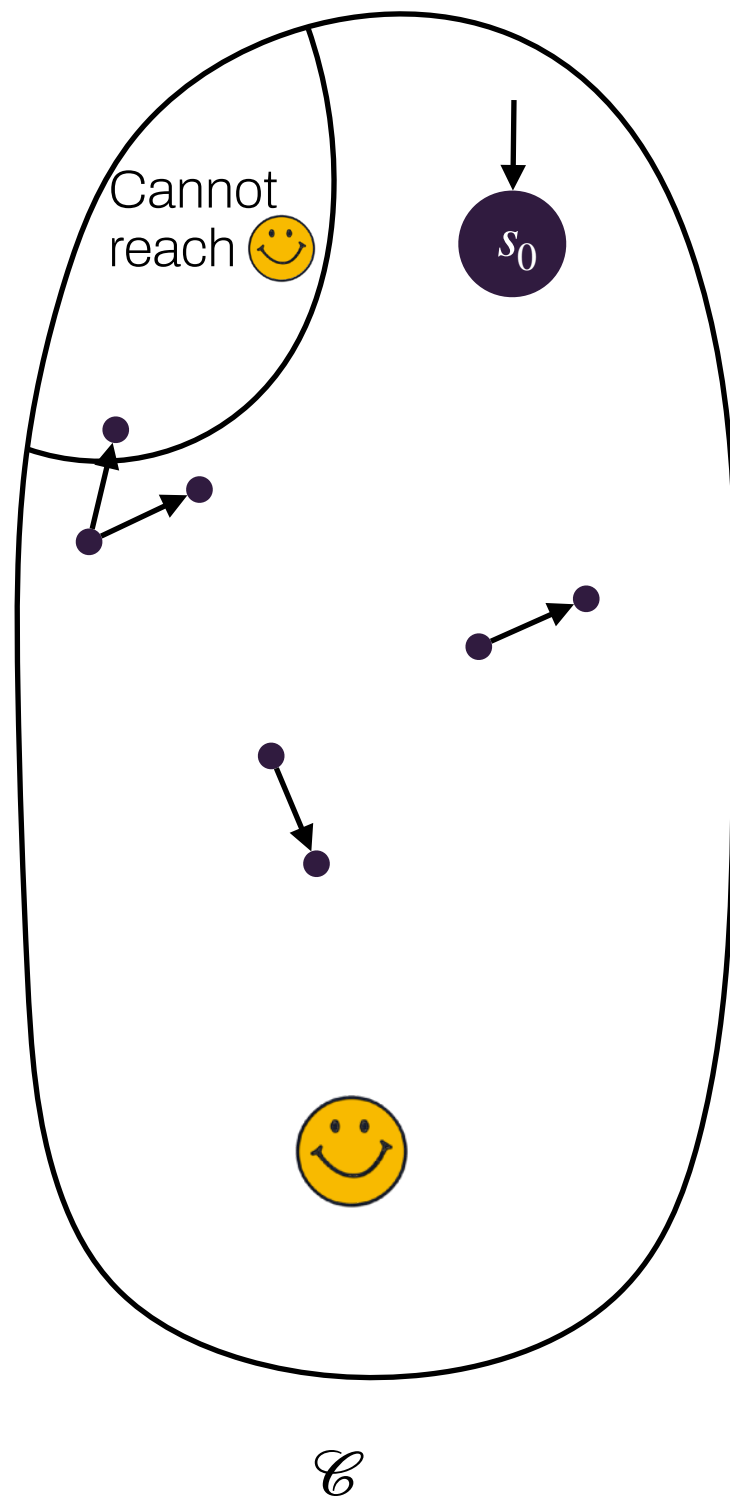
Termination, efficiency and guarantees

Termination

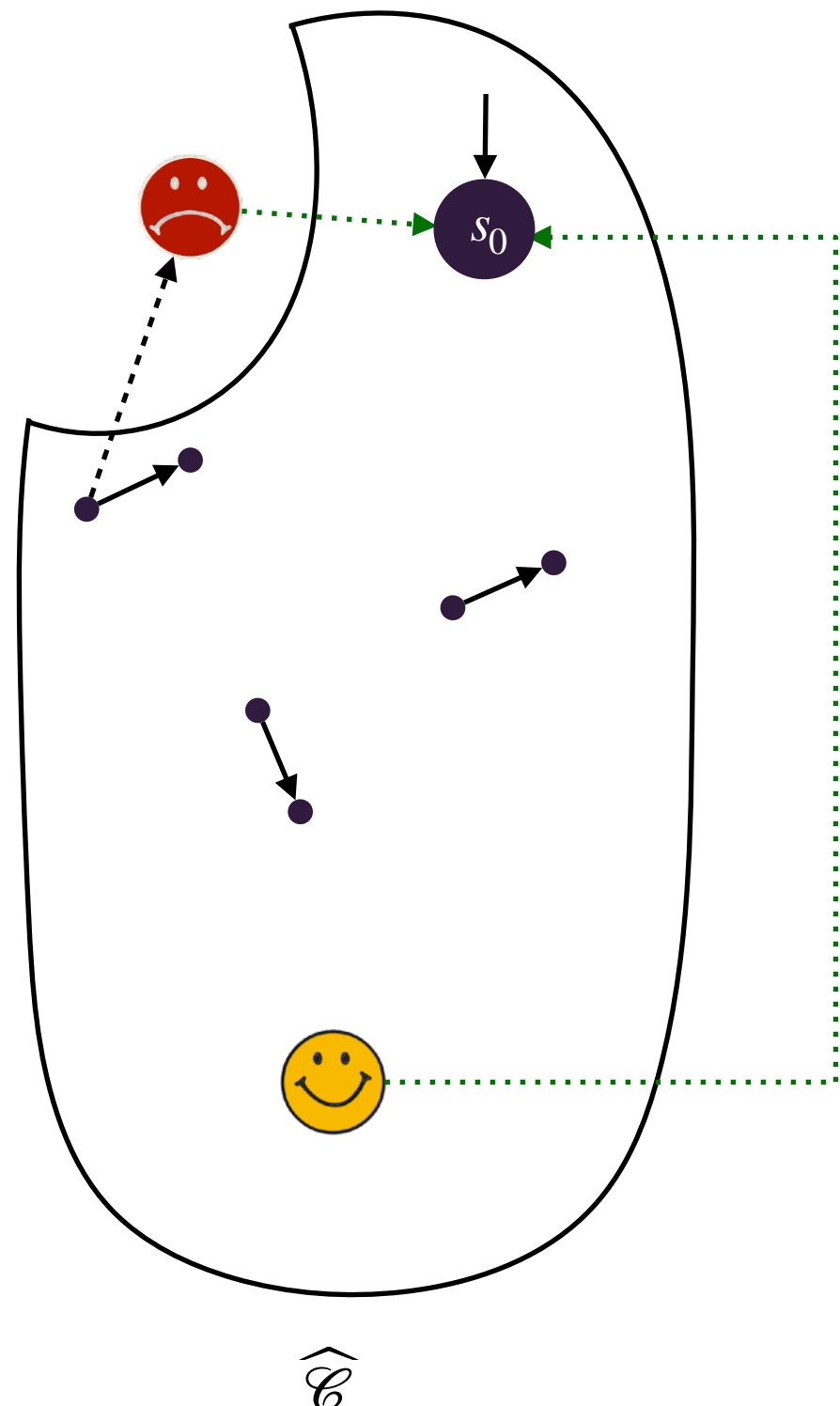
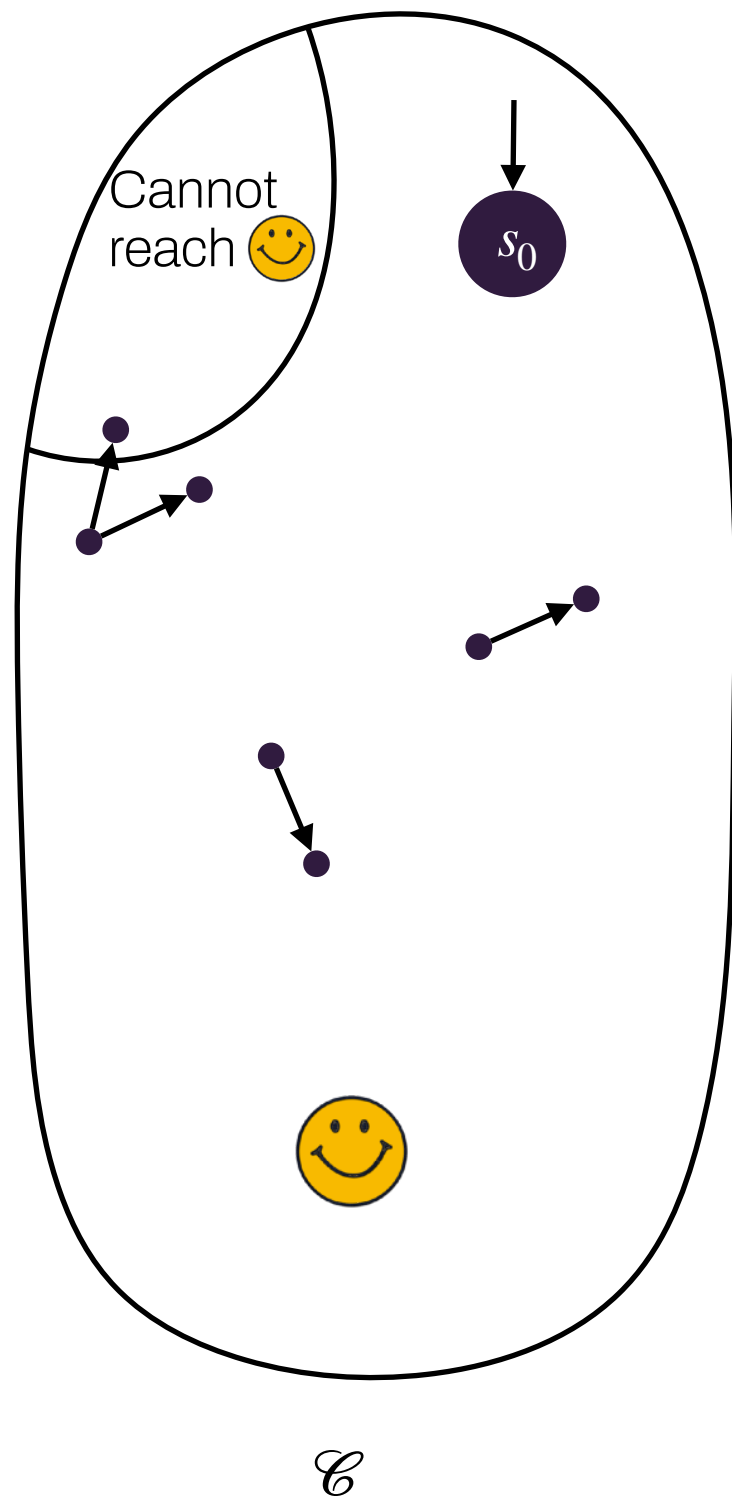
(To our knowledge, never expressed like this)

A sampled path starting at s_0 almost-surely hits 😊 or 😞
iff
 $\mathcal{C}$ is decisive from s_0 w.r.t. 😊

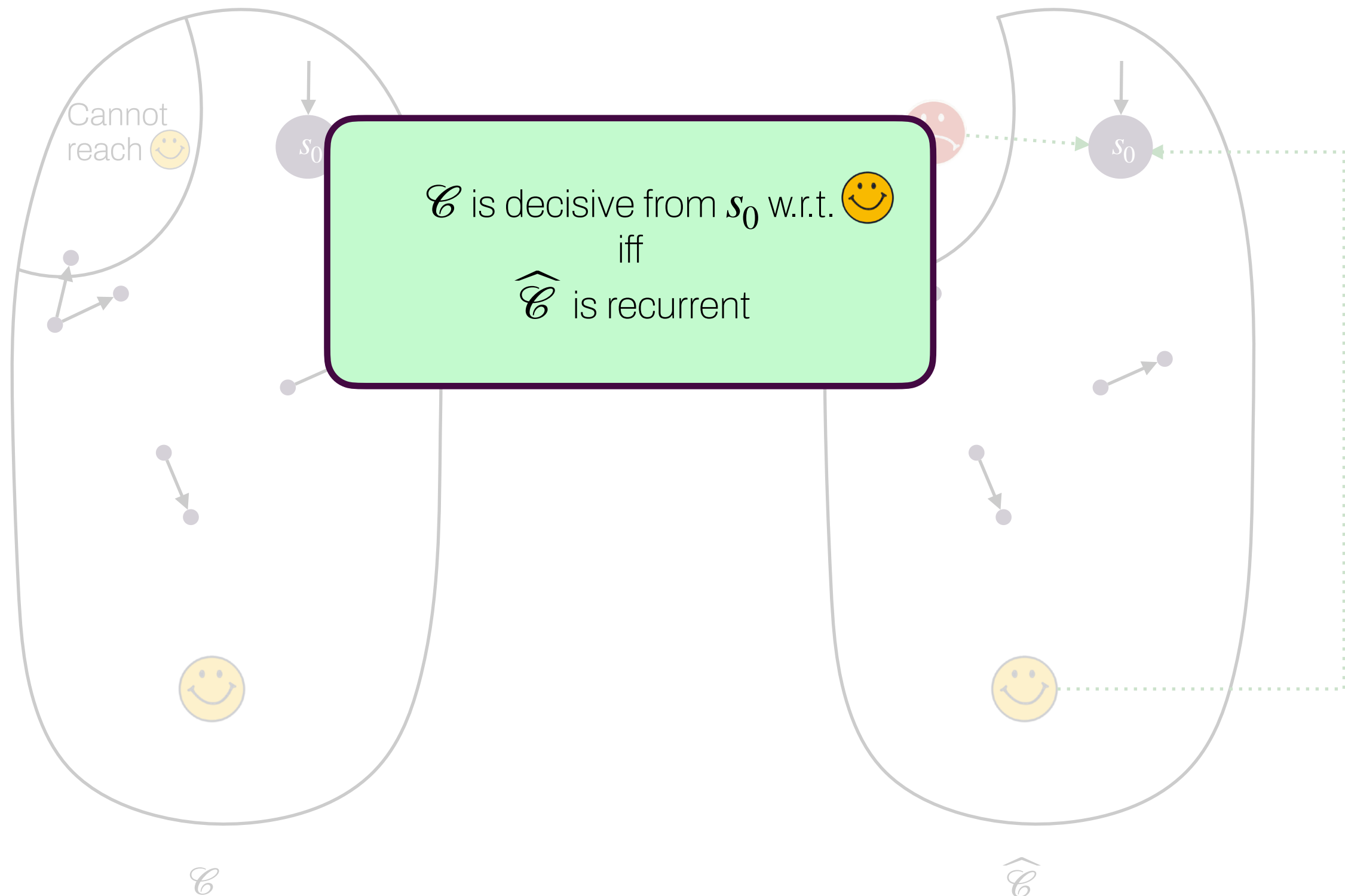
Decisiveness vs recurrence



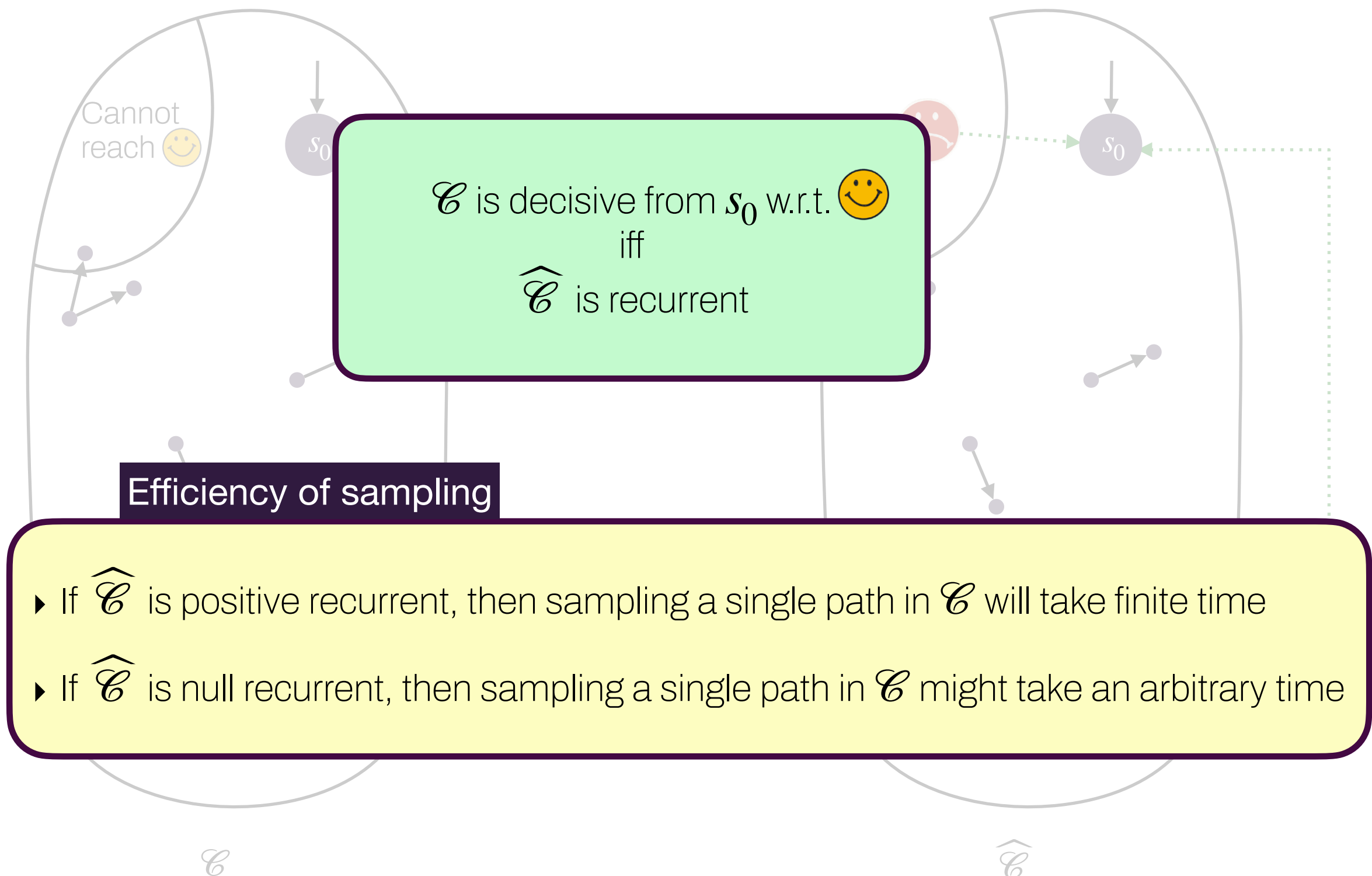
Decisiveness vs recurrence



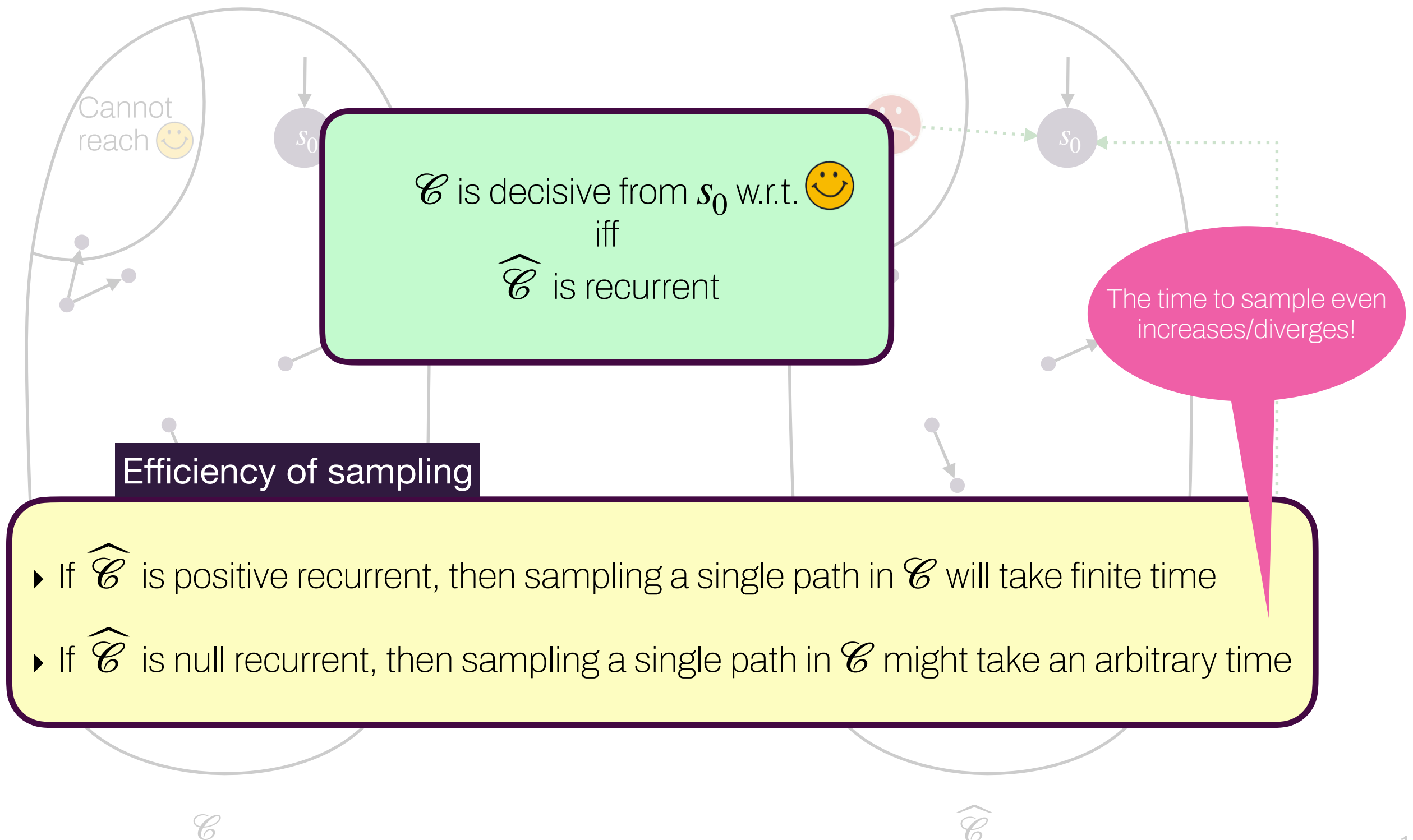
Decisiveness vs recurrence



Decisiveness vs recurrence



Decisiveness vs recurrence



Termination, efficiency and guarantees

Termination

(To our knowledge, never expressed like this)

A sampled path starting at s_0 almost-surely hits 😊 or 😞
iff

$\mathcal{C}$ is decisive from s_0 w.r.t. 😊

+ efficiency if finite return time
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Guarantees: Hoeffding's inequalities

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$$\mathbb{P}\left(\left|\frac{n_N}{N} - \mathbb{P}(\mathbf{F} \text{ 😊})\right| \geq \frac{\varepsilon}{2}\right) \leq \delta$$

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Confidence level

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Confidence level

Precision

$\left[\frac{n_N}{N} - \frac{\varepsilon}{2}; \frac{n_N}{N} + \frac{\varepsilon}{2}\right]$: confidence interval

Termination, efficiency and guarantees

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Empirical estimation

Let $\varepsilon, \delta > 0$ s.t. $N \geq \frac{8B^2}{\varepsilon^2} \log\left(\frac{2}{\delta}\right)$. Then:

B bound on the function

$$\mathbb{P}\left(\left|\frac{f_N}{N} - \mathbb{E}(f_L, \text{😊})\right| \geq \frac{\varepsilon}{2}\right) \leq \delta$$

$\left[\frac{f_N}{N} - \frac{\varepsilon}{2}; \frac{f_N}{N} + \frac{\varepsilon}{2}\right]$: confidence interval

Value given by L for paths that stop at 😊

What can we do for
non-decisive Markov chains??

Importance sampling [KH51]

- ▶ Originally used for rare events

Rare events problem

- Issue: rare events in $\mathcal{C}$

Rare-Event Problem for Statistical Model Checking

Problem Statement

- We want to estimate the probability of a rare event e occurring with probability close to 10^{-15} .
- We want a *confidence level* of 0.99.
- We are able to compute 10^9 trajectories.

Possible Outcomes

Number of occurrences of e	Probability	Confidence interval
0	$\approx 1 - 10^{-6}$	$[0, 7.03 \cdot 10^{-9}]$
1	$\leq 10^{-6}$	$[6.83 \cdot 10^{-10}, 1.69 \cdot 10^{-9}]$
$n > 1$	$\leq 10^{-12}$	$> 6.83 \cdot 10^{-10}$

Importance sampling

[KH51]

[KH51] H. Kahn, T. E. Harris. *Estimation of particle transmission by random sampling* (National Bureau of Standards applied mathematics series, 1951)

[Bar14] B. Barbot. *Acceleration for statistical model checking* (PhD thesis)

[BHP12] B. Barbot, S. Haddad, C. Picaronny. *Coupling and Importance Sampling for Statistical Model Checking* (TACAS'12)

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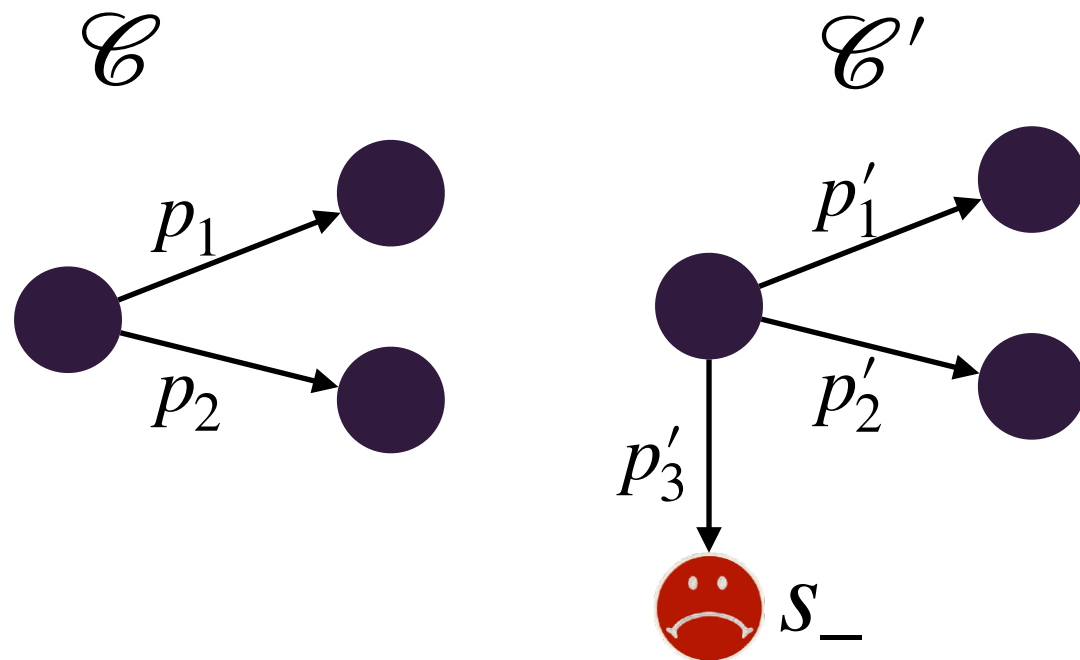
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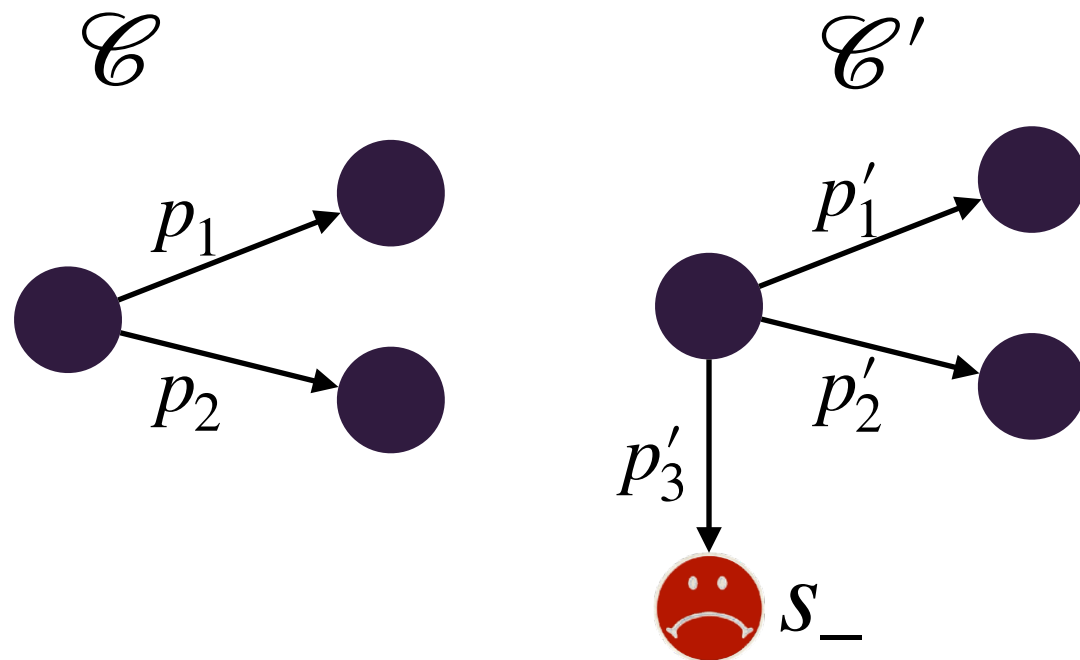
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Correct the bias

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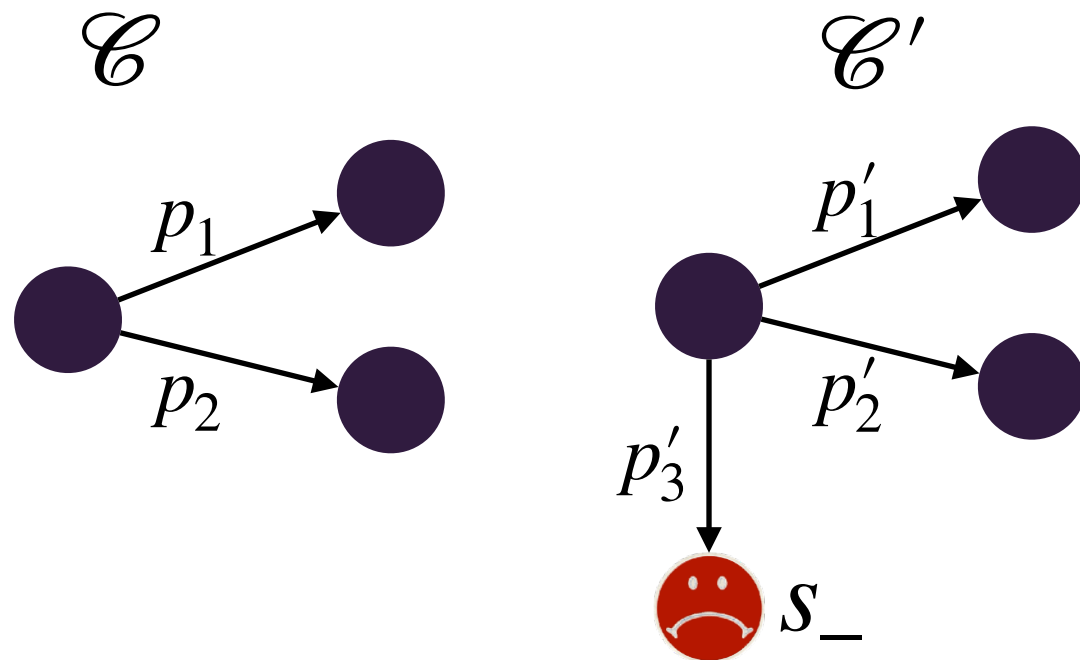
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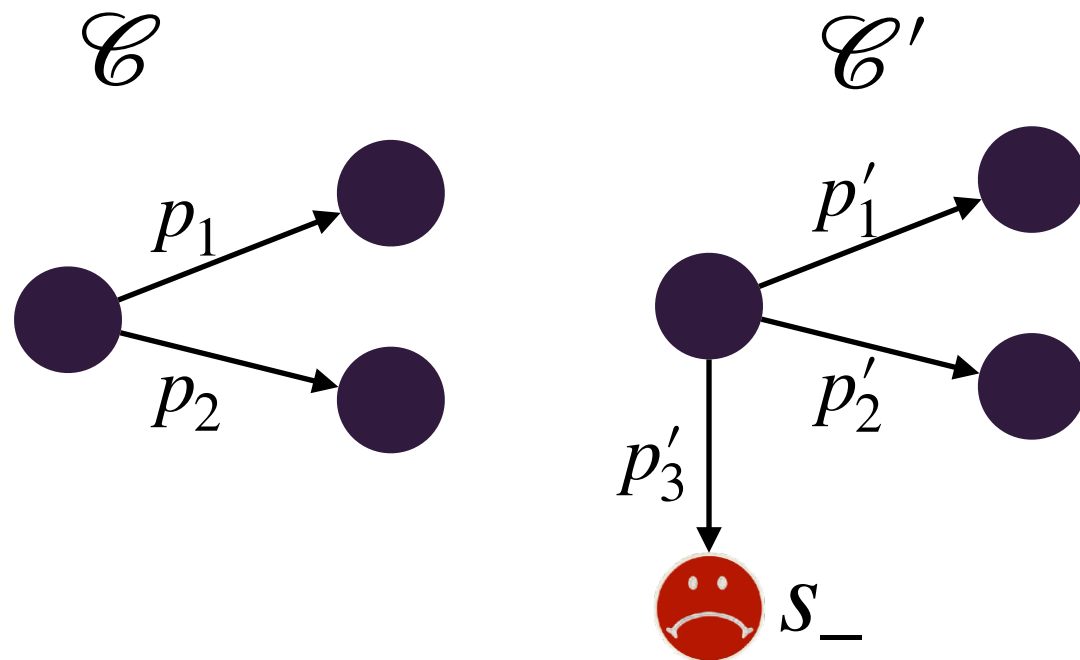
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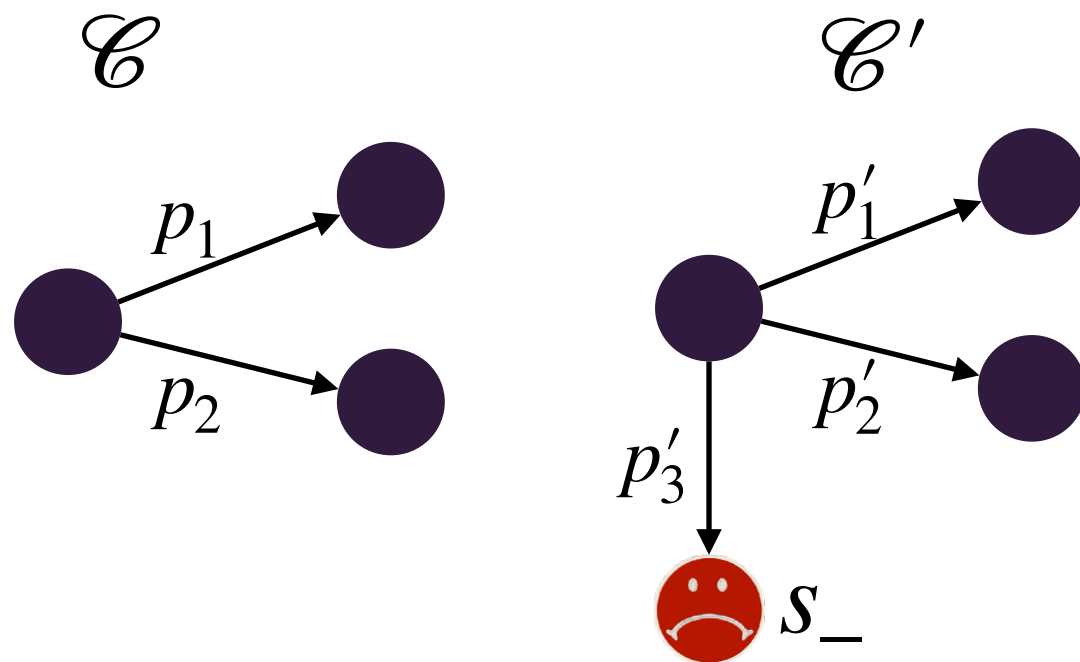
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- Originally used for rare events
- Setting giving statistical guarantees [BHP12, Bar14]

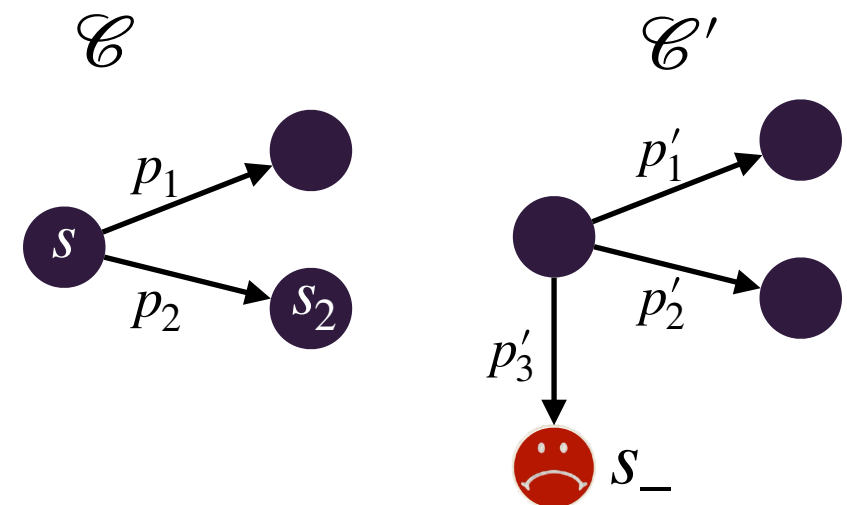
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Properties of the biased Markov chain

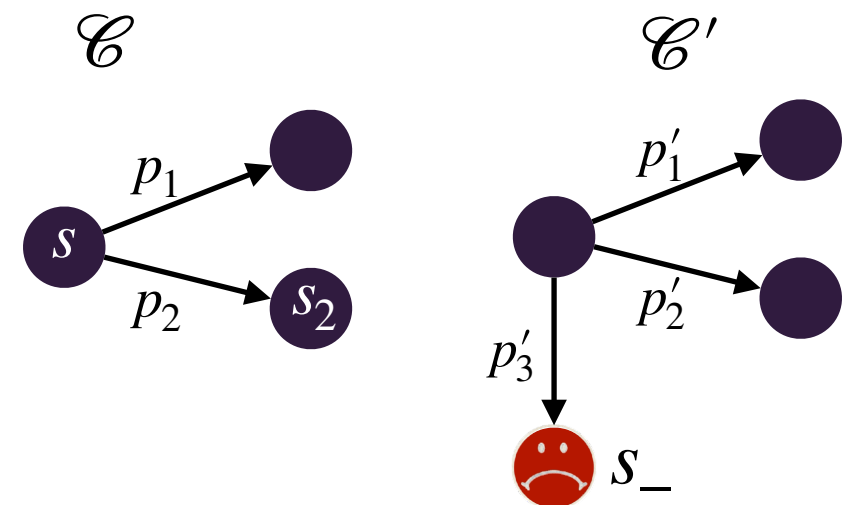
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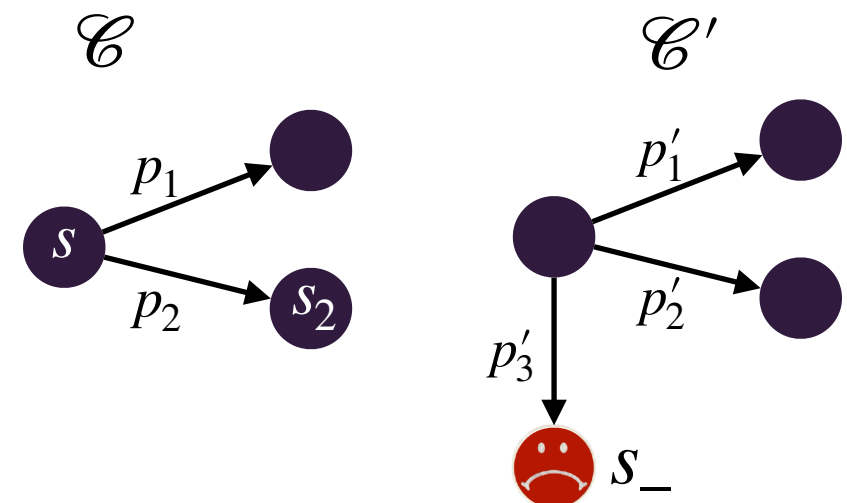
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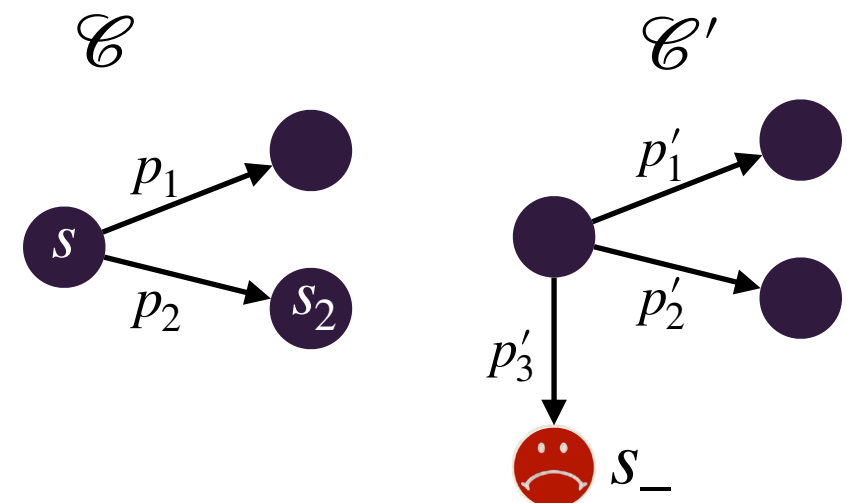
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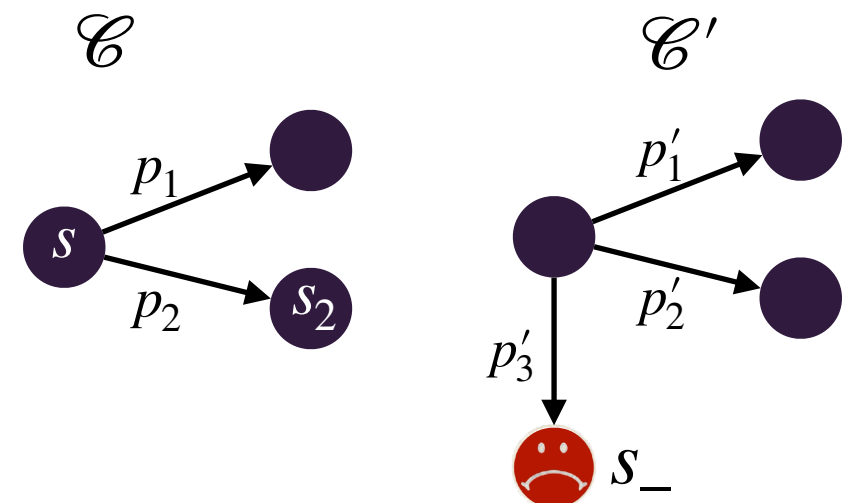


Properties of the biased Markov chain

Define $\mu(s)$ as
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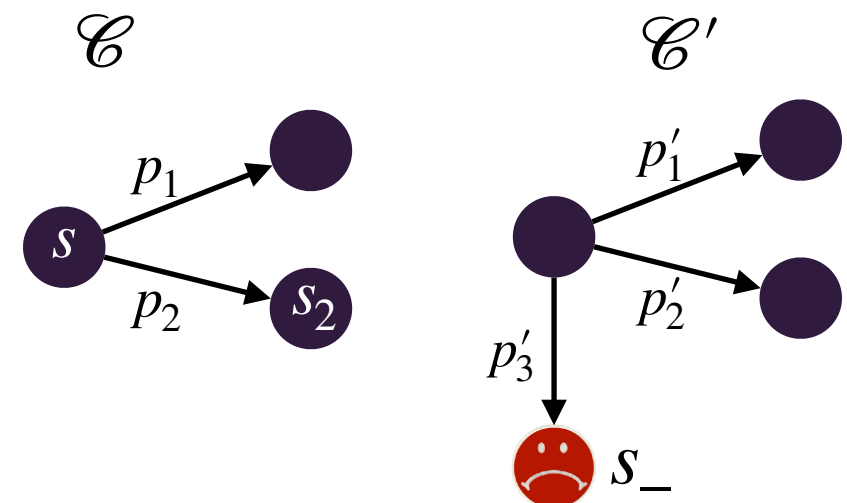
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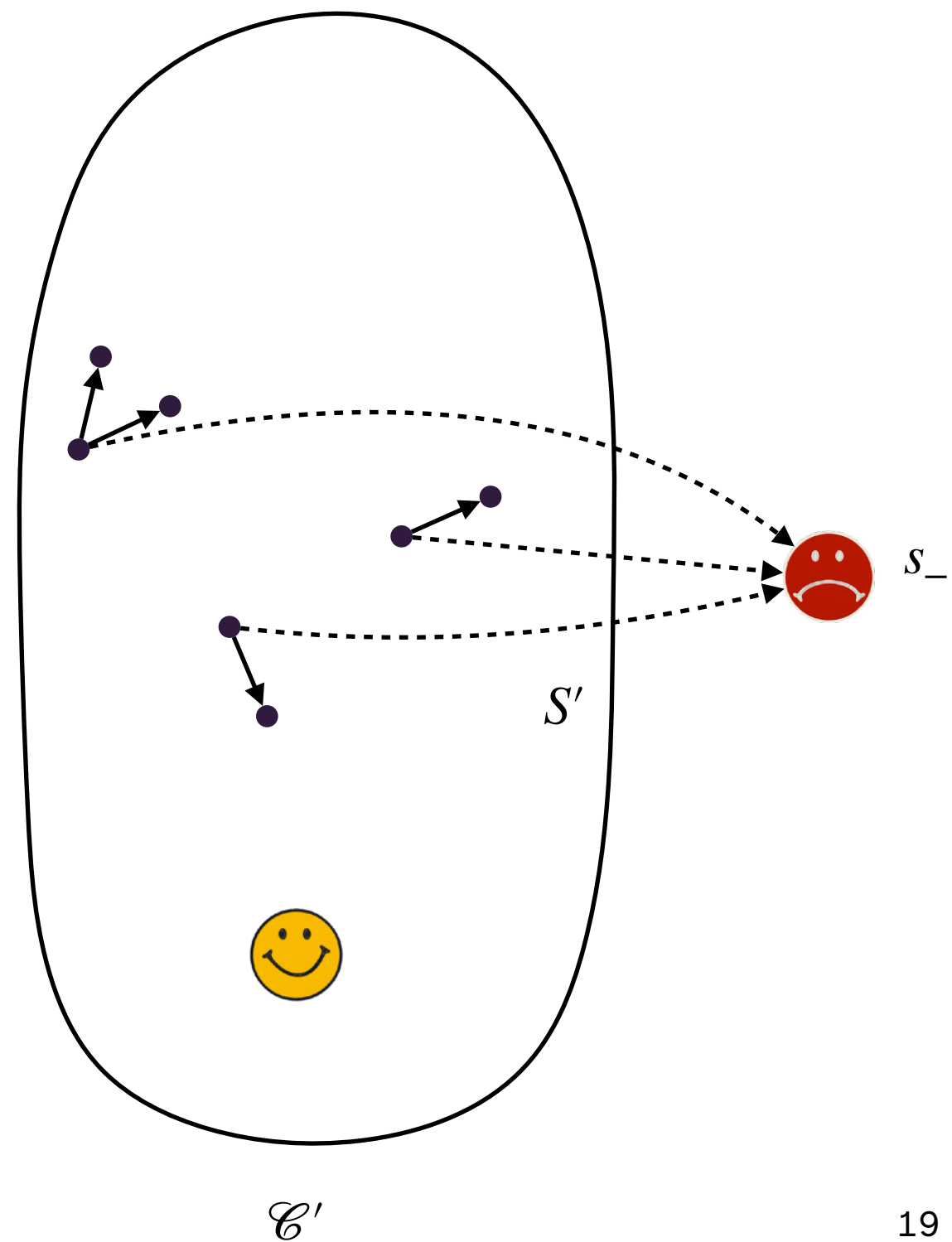
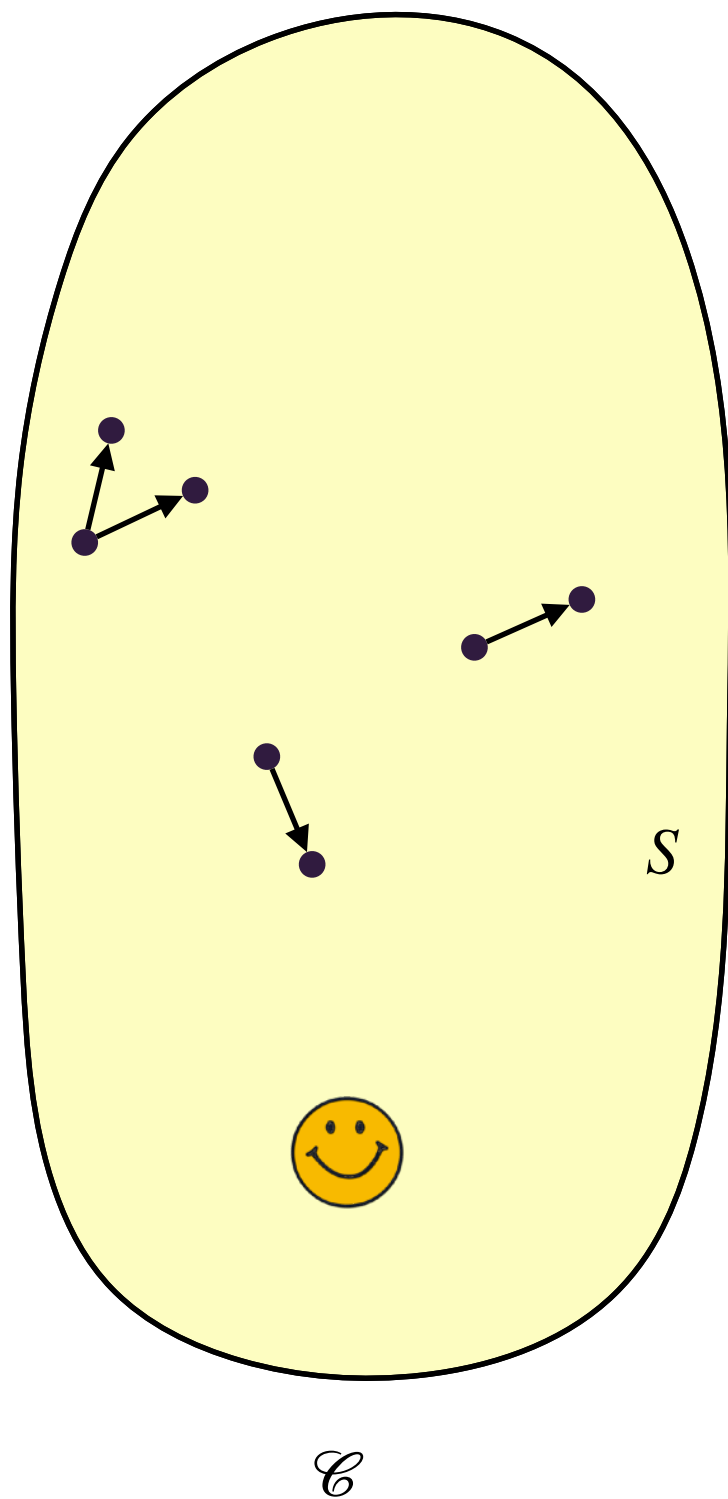
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There is a best choice: $p'_i = \frac{\mu(s_i)}{\mu(s)} \cdot p_i$

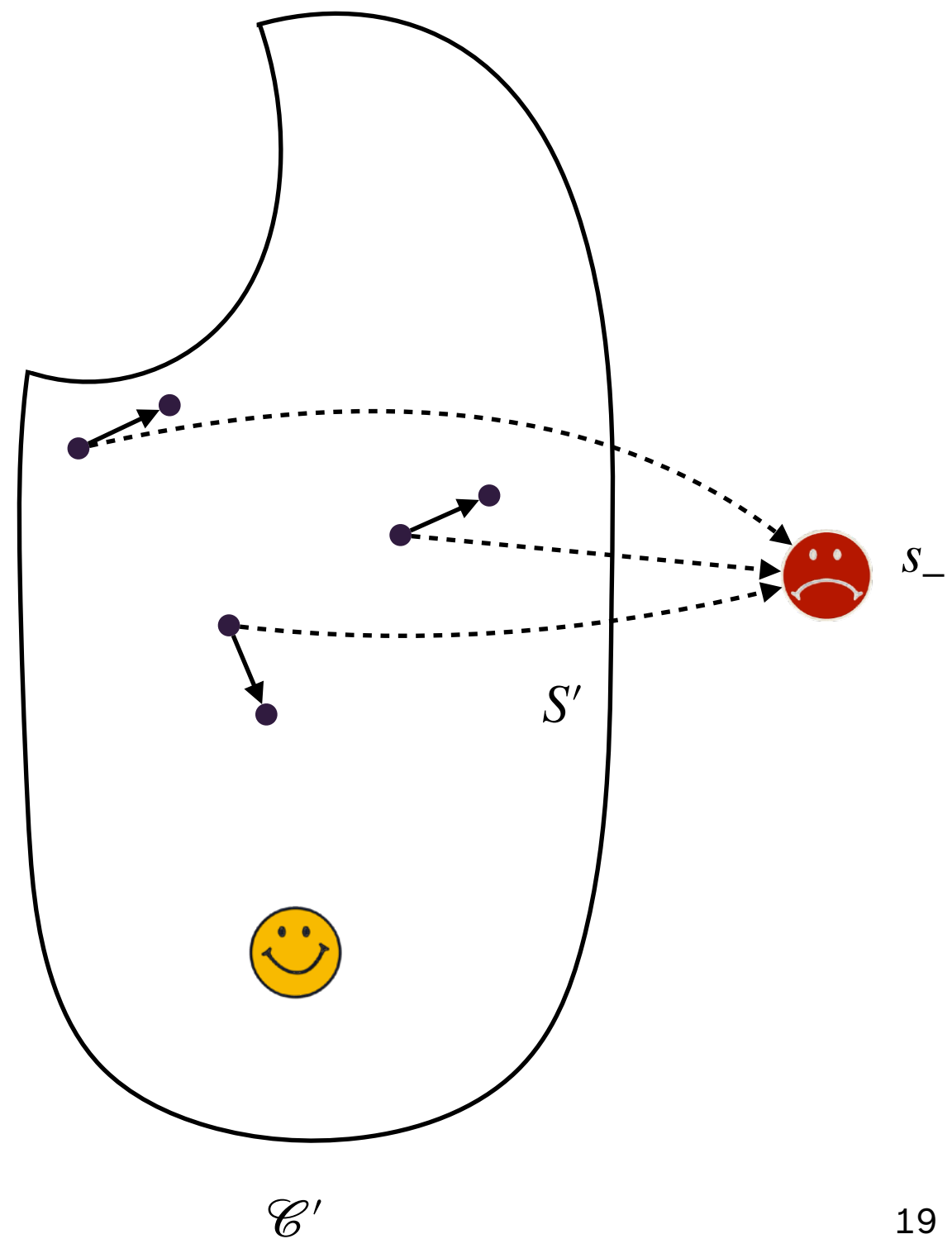
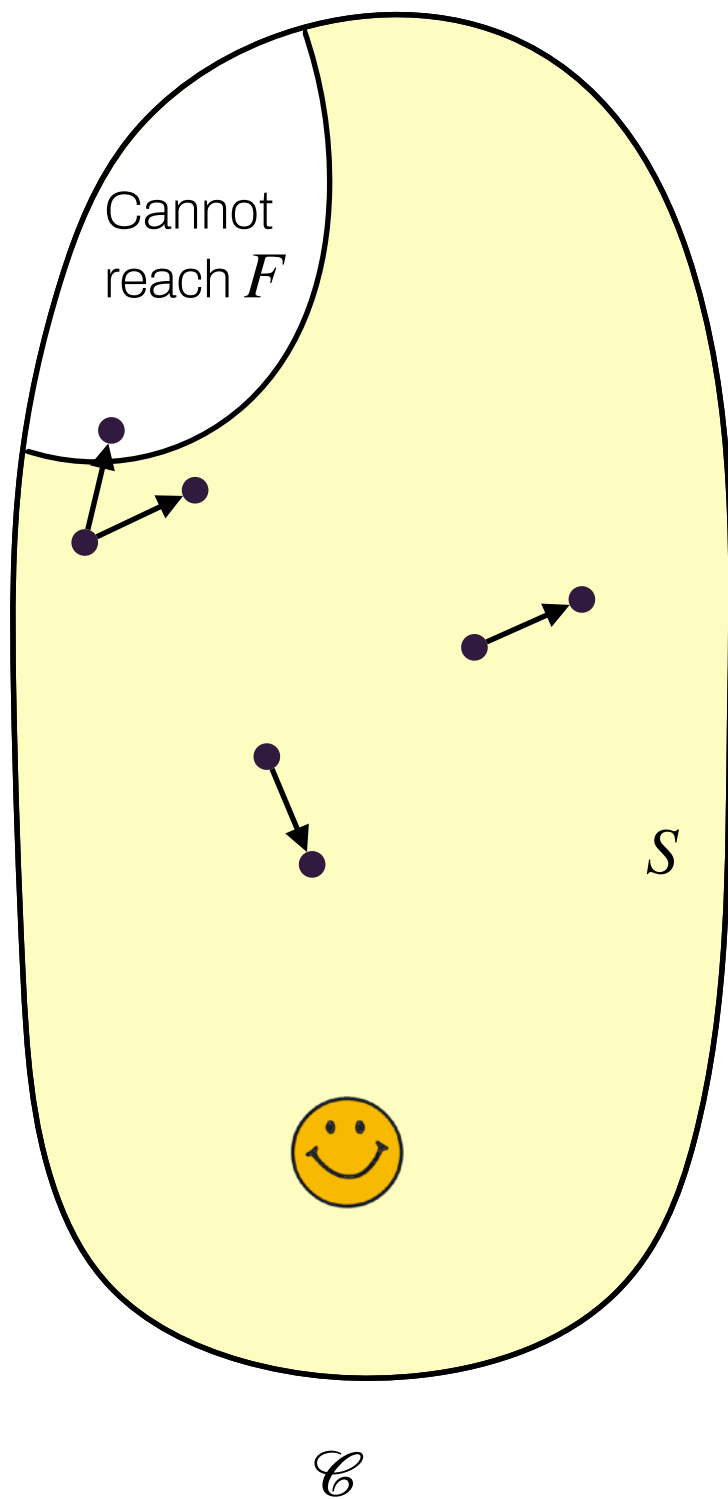
- ▶ The r.v. in $\mathcal{C}'$ takes value $\mu(s)$
- ▶ One needs to know μ !



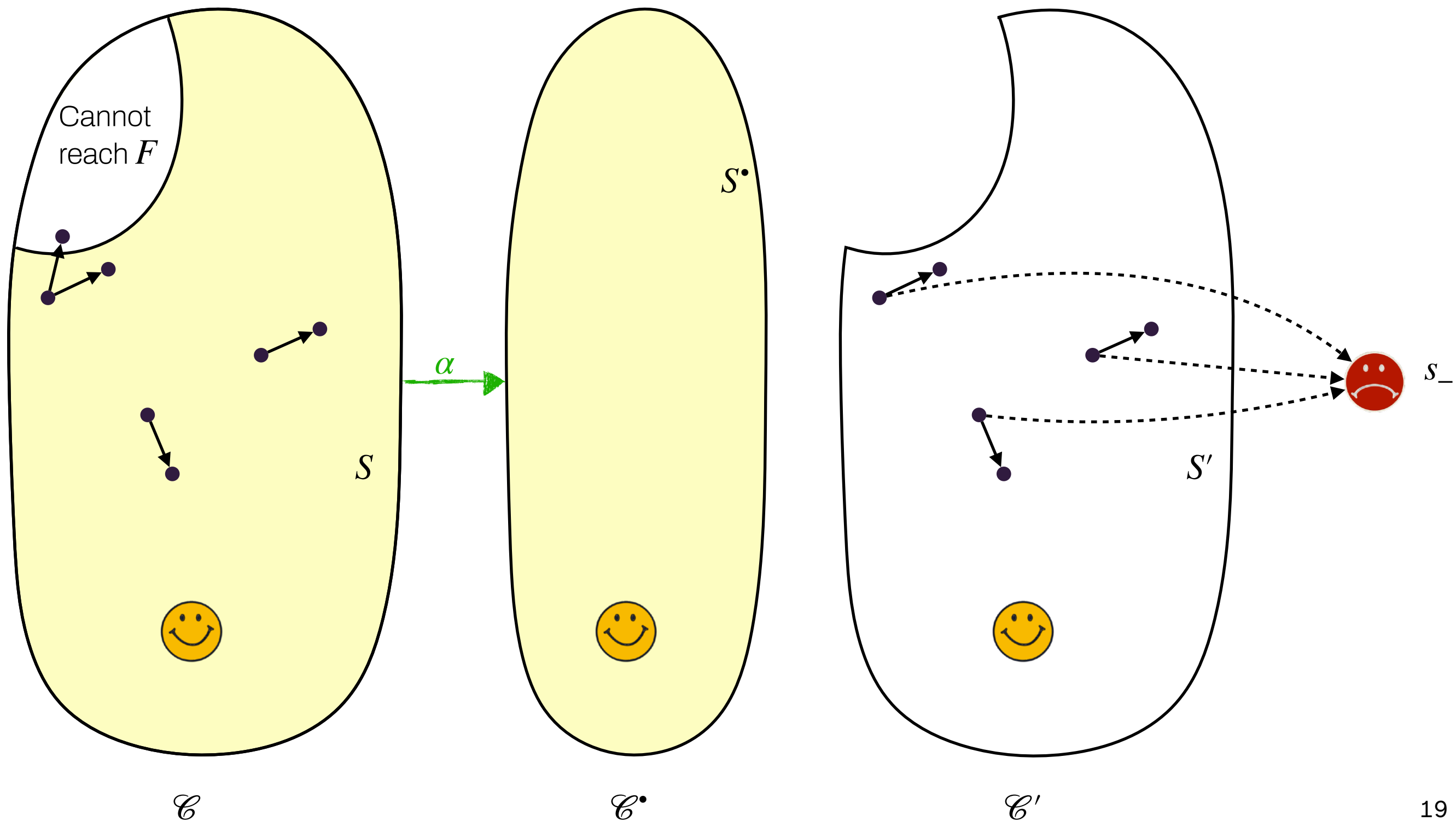
Importance sampling via an abstraction



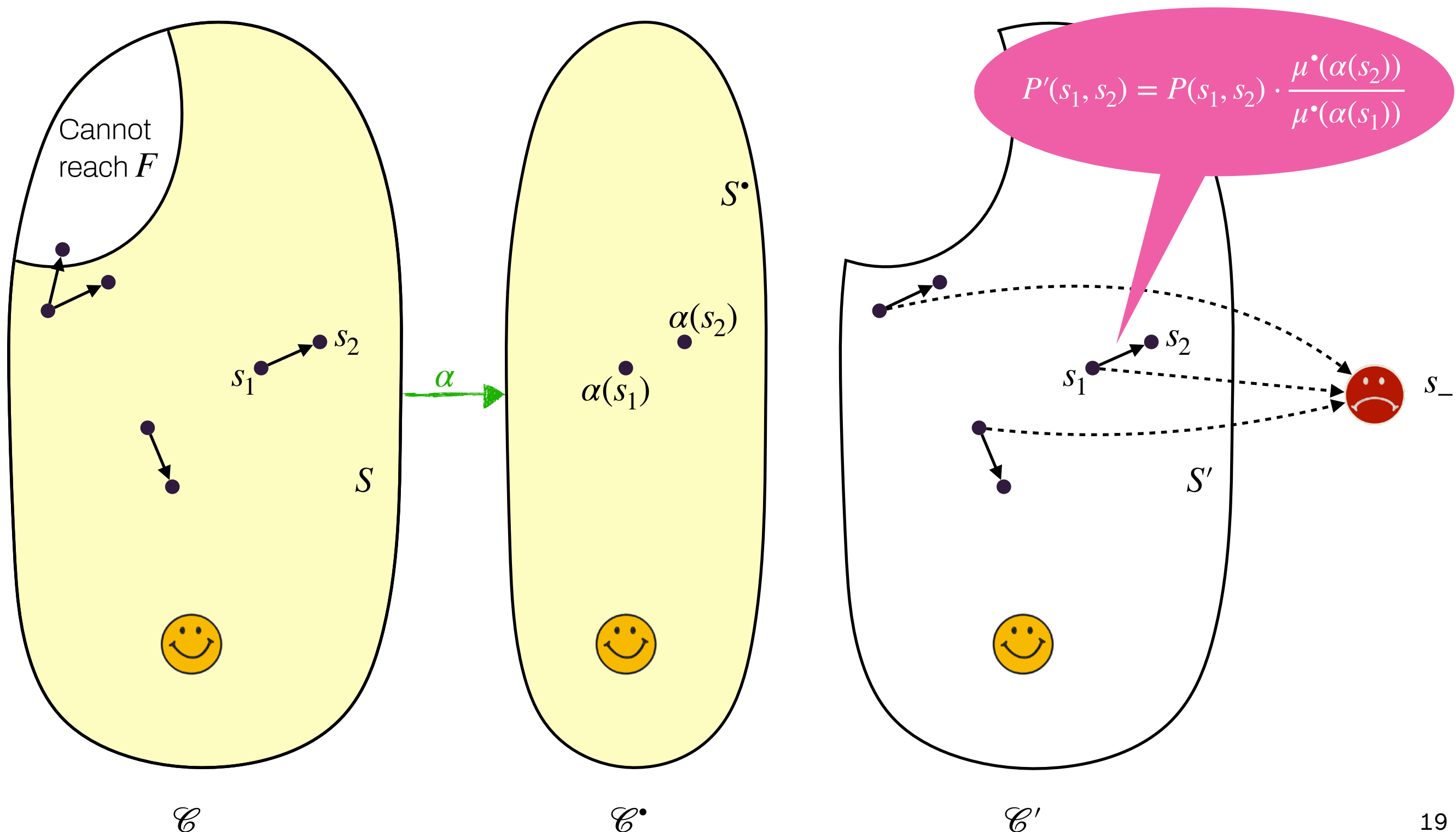
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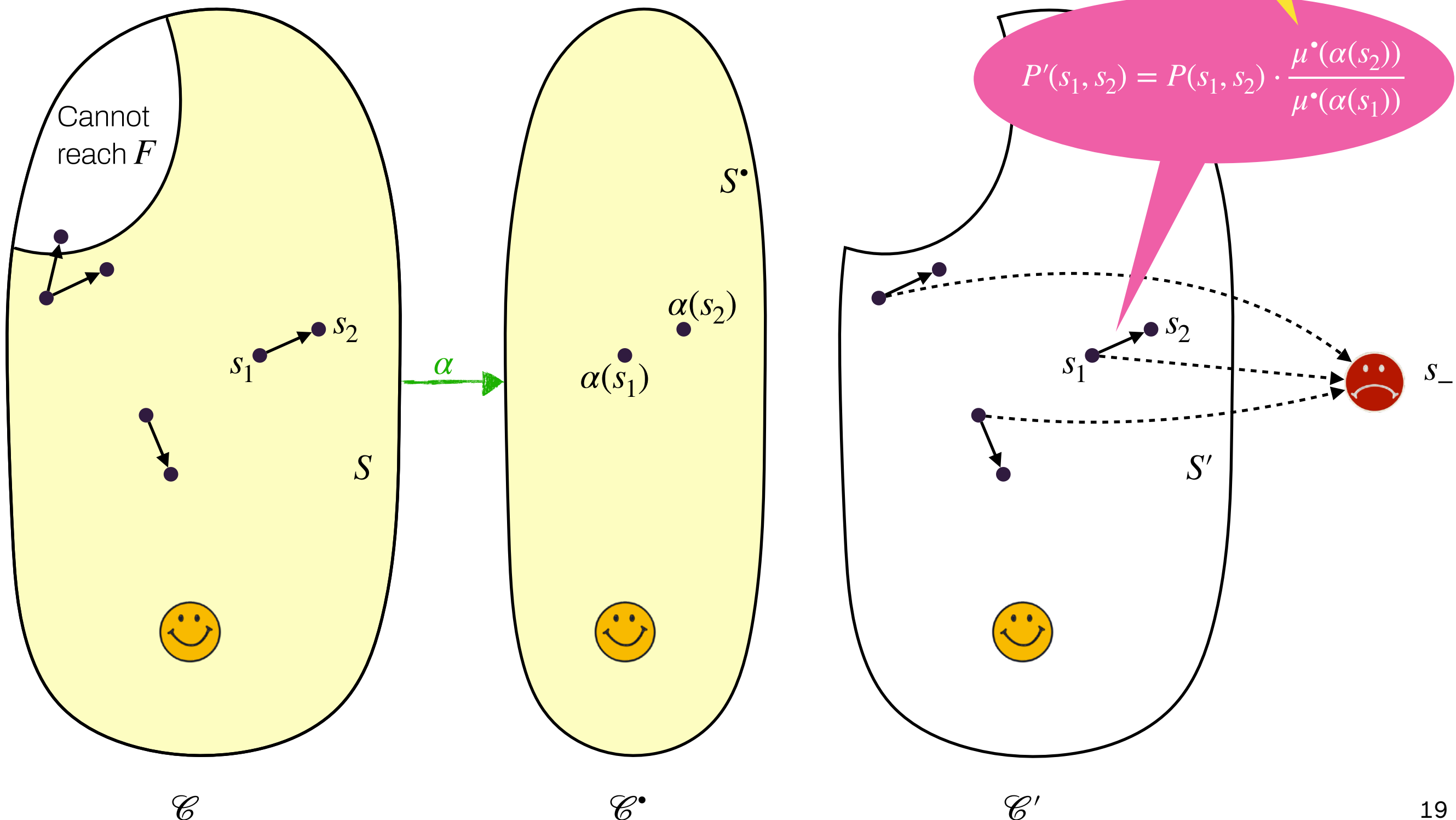


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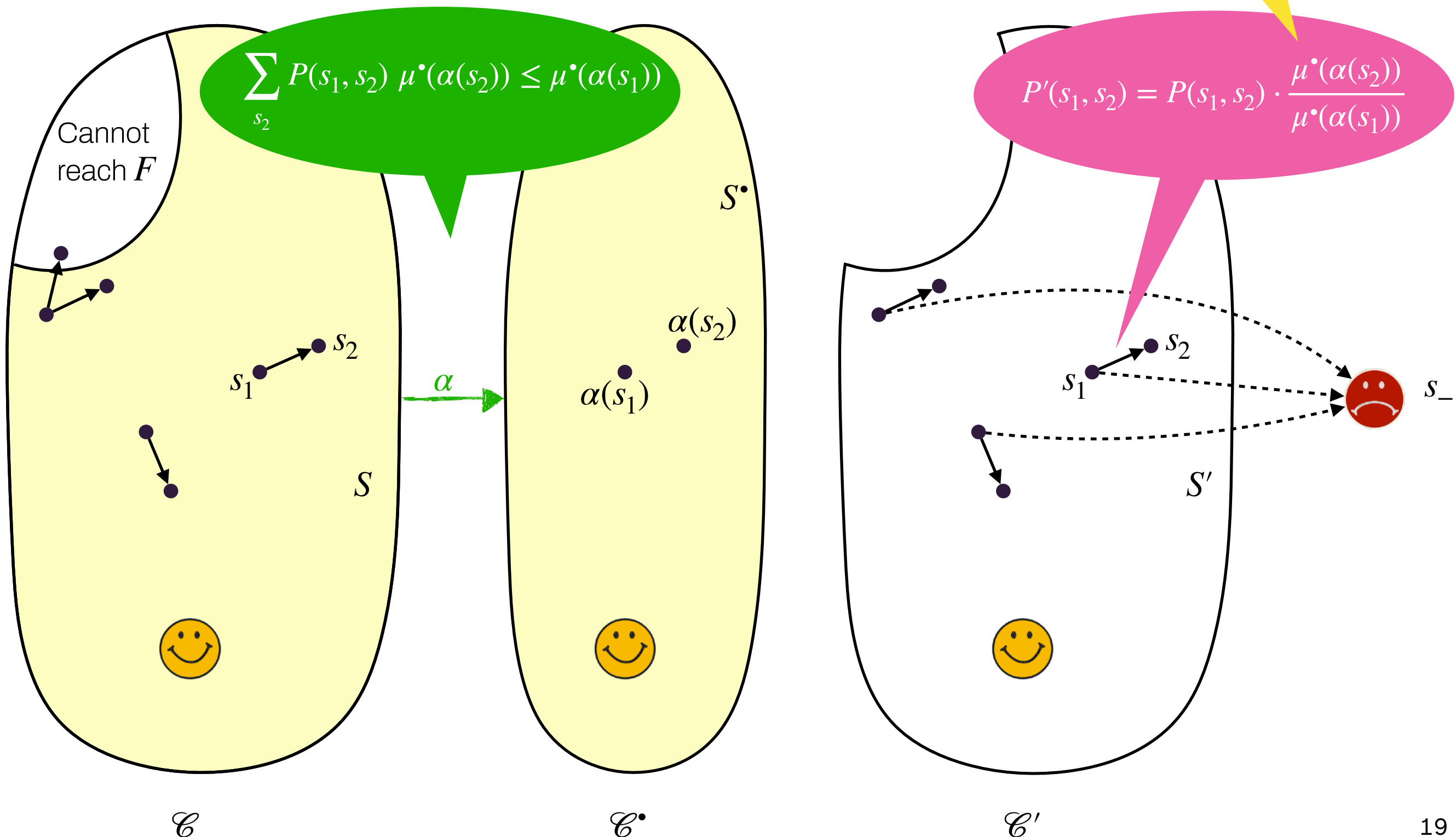


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$\mu^\bullet$ is the probability
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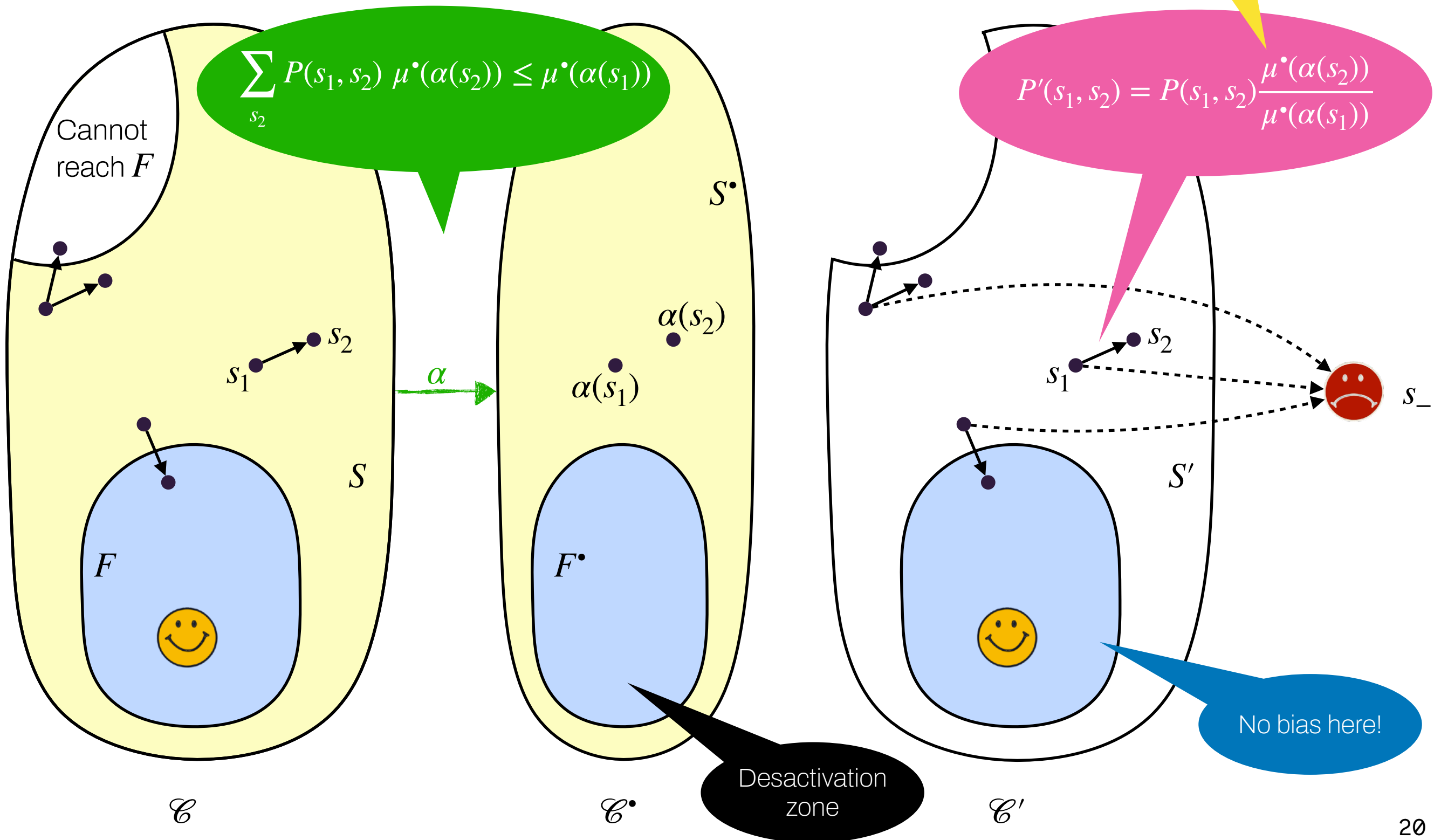


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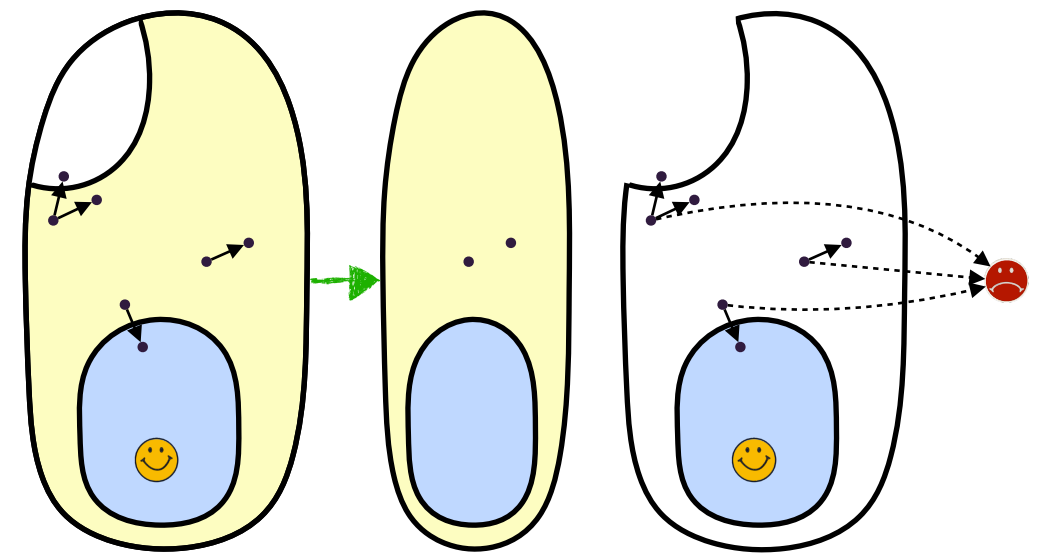


Importance sampling via an abstraction

$\mu^\bullet$ is the probability to reach $F^\bullet$ in $\mathcal{C}^\bullet$



Further properties of the biased Markov chain obtained via an abstraction

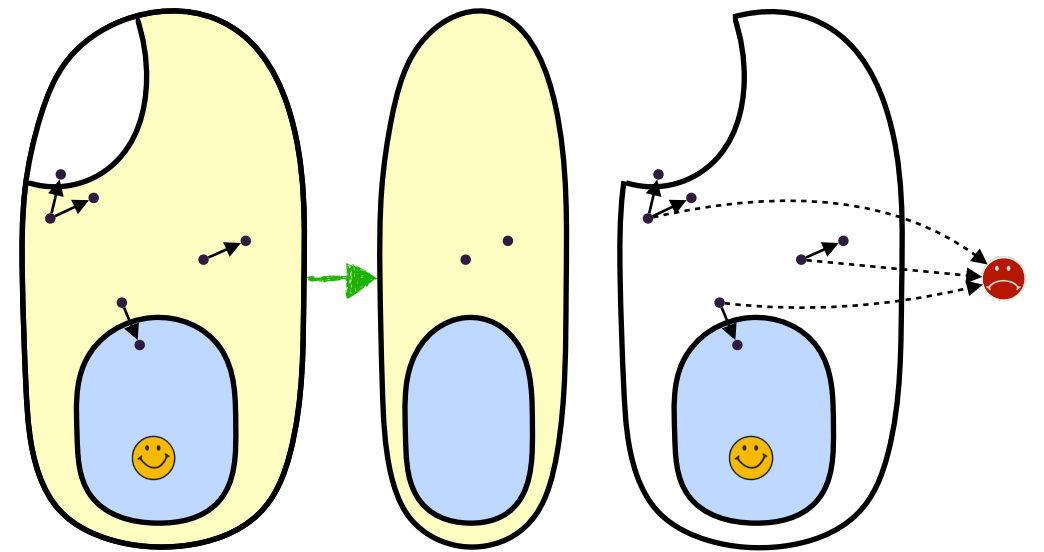


Further properties of the biased Markov chain obtained via an abstraction

Property of the bias

$$\gamma(\rho) = \begin{cases} \mu^\bullet(\alpha(s_0)) & \text{if } \rho \text{ ends in } \text{😊} \\ 0 & \text{otherwise} \end{cases}$$

It is bivaluated, hence bounded

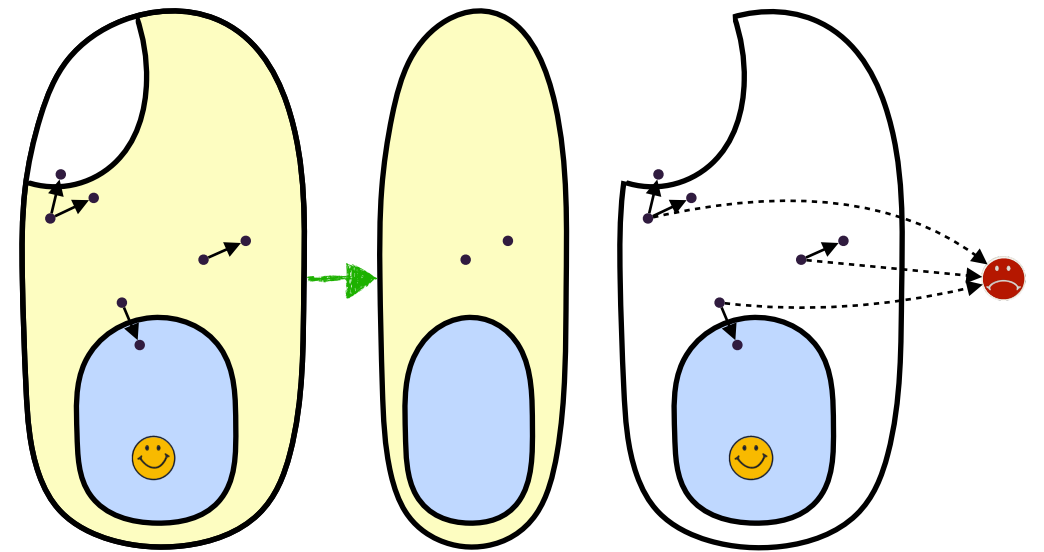


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Theorem

If F is finite and for every $0 \leq x < 1$, $\{s \in S \mid \mu^\bullet(\alpha(s)) \geq x\}$ is finite, then $\mathcal{C}'$ is decisive w.r.t. 😊.

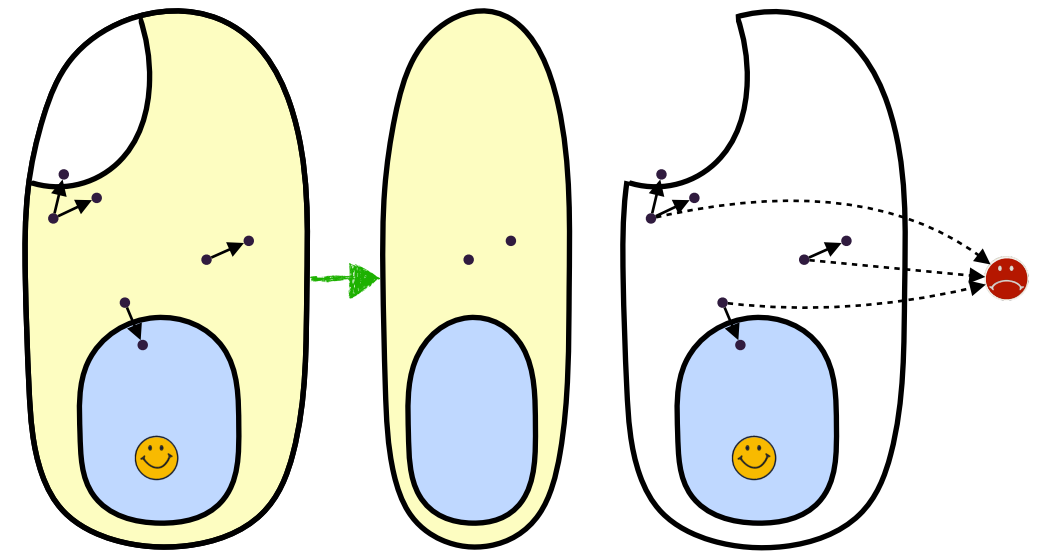
Proof using attractors, martingale theory

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- The analysis can be performed on $\mathcal{C}'$!

And «concretely»?

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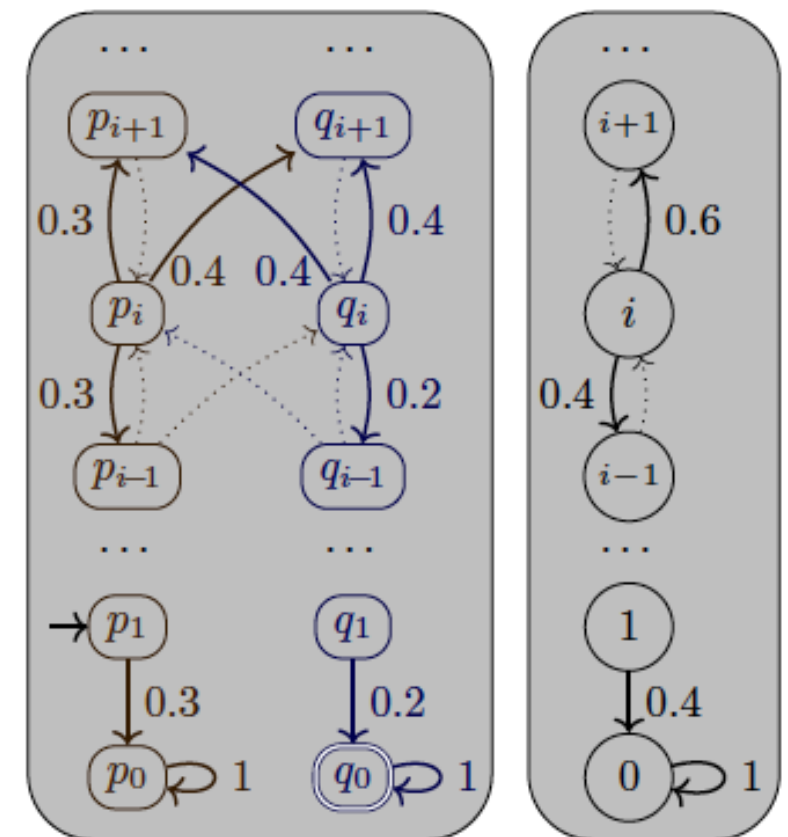
- ▶ Model = layered Markov chain (LMC) $\mathcal{C}$: there is a level function $\lambda : S \rightarrow \mathbb{N}$ s.t.
 - for every $s_1 \rightarrow s_2$, $\lambda(s_1) - \lambda(s_2) \leq 1$, and
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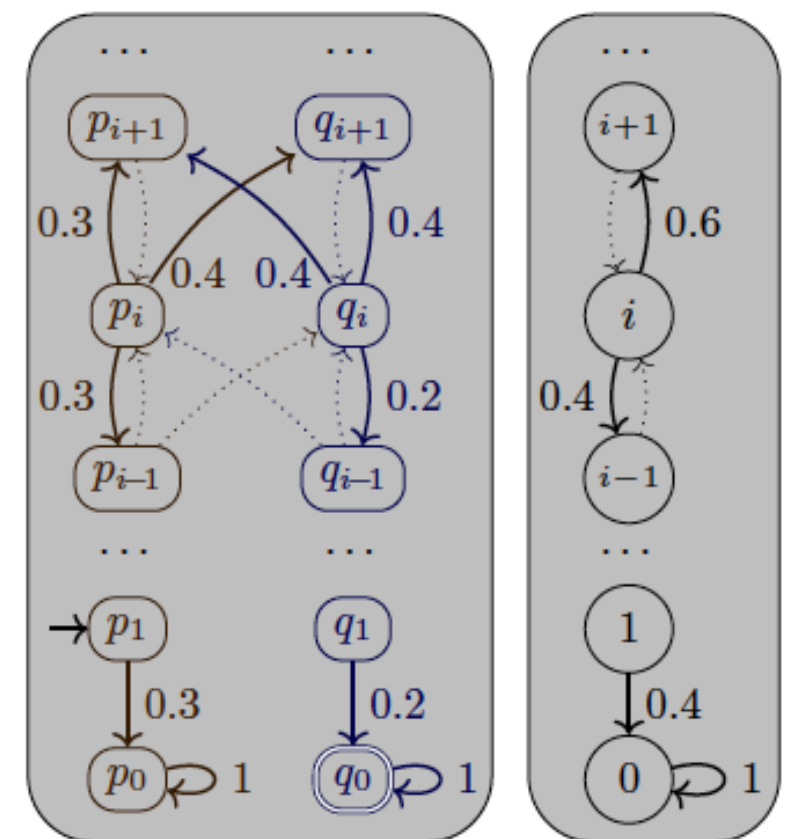
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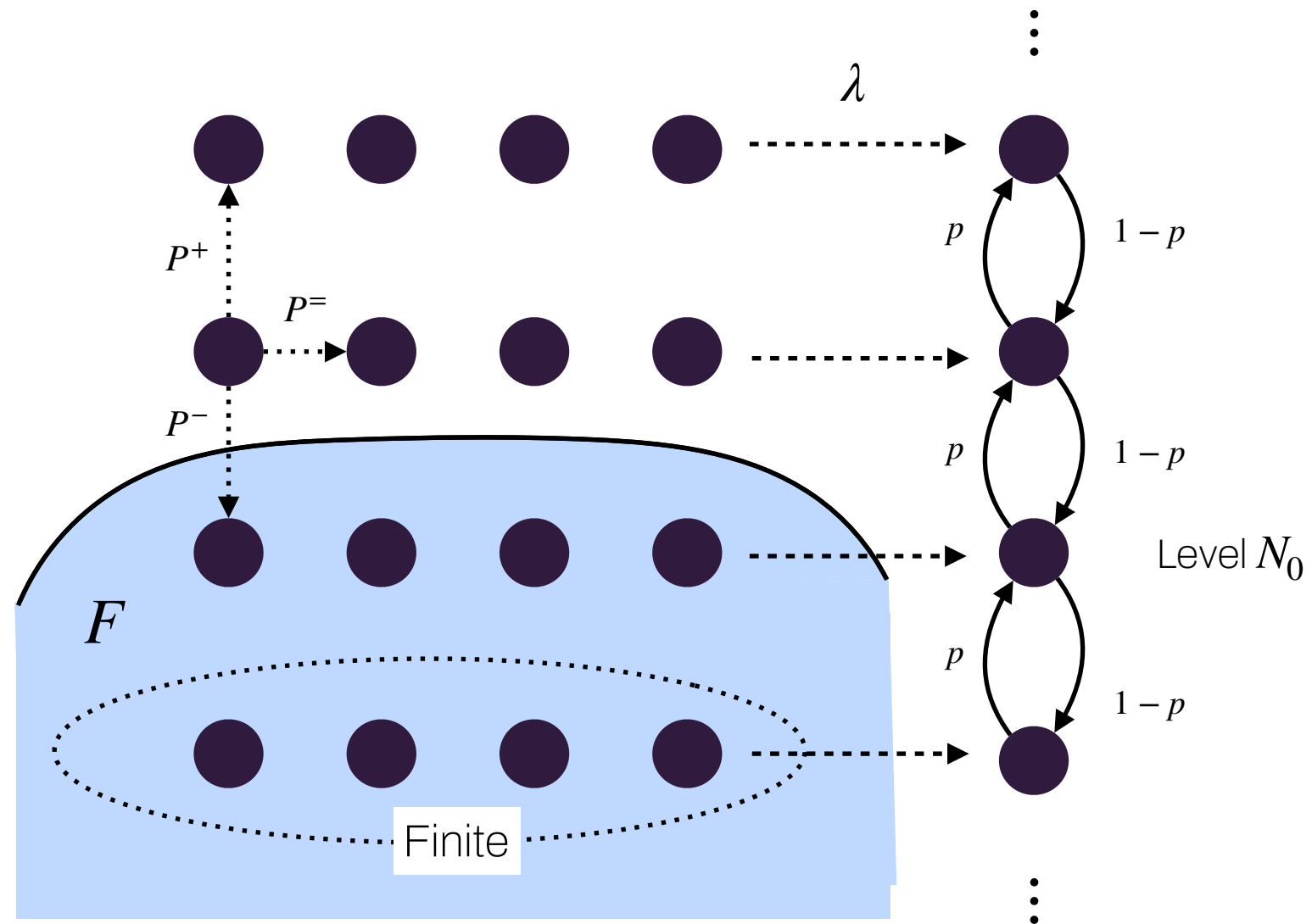
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Only one condition needs to be satisfied...
The monotony condition!



Condition for monotony

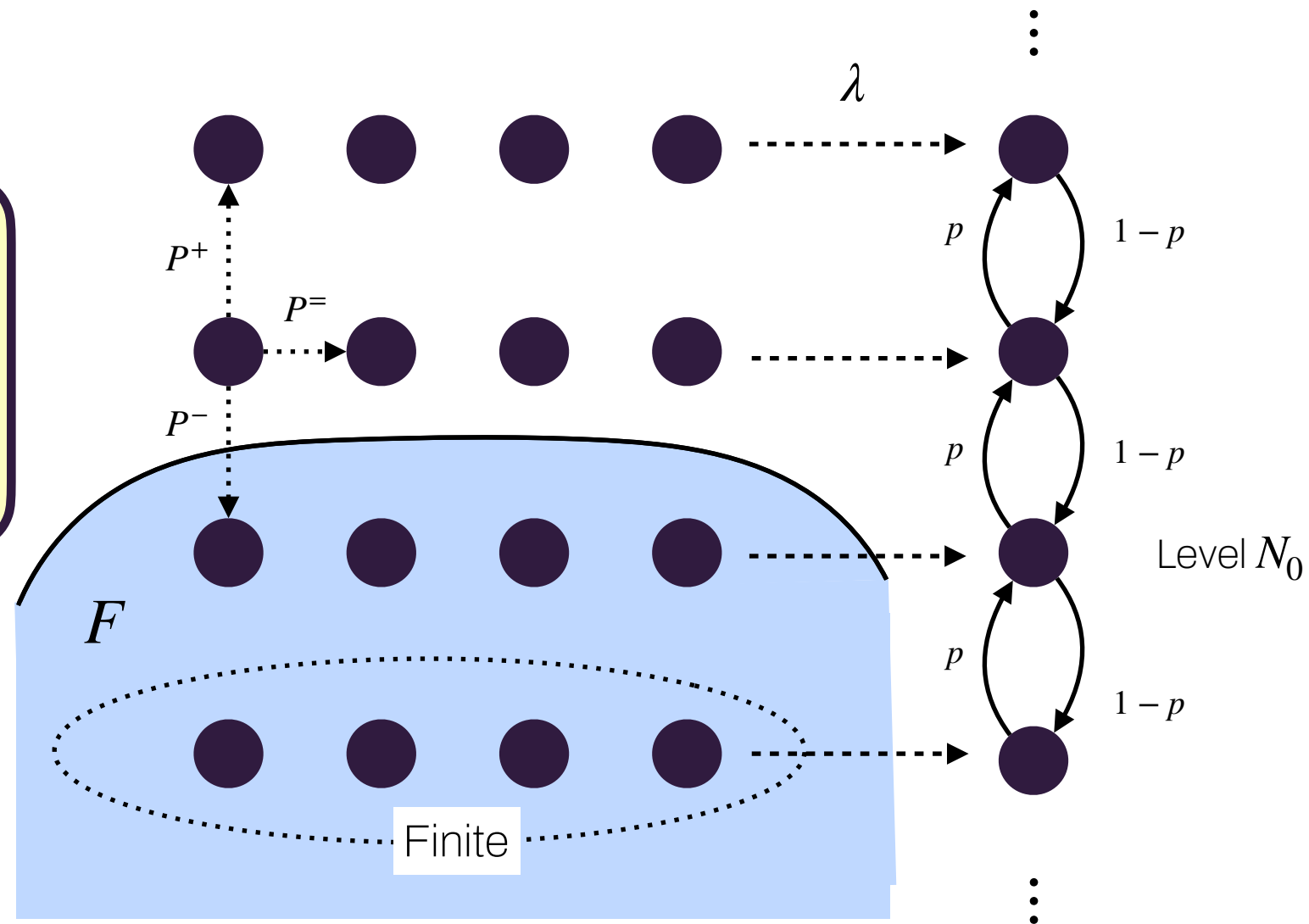


Condition for monotony

$(\hat{p}, N_0)$ -divergence

For every $s \notin F$, $P^-(s) < 1$ implies

$$\frac{P^+(s)}{P^-(s) + P^+(s)} \geq \hat{p}$$



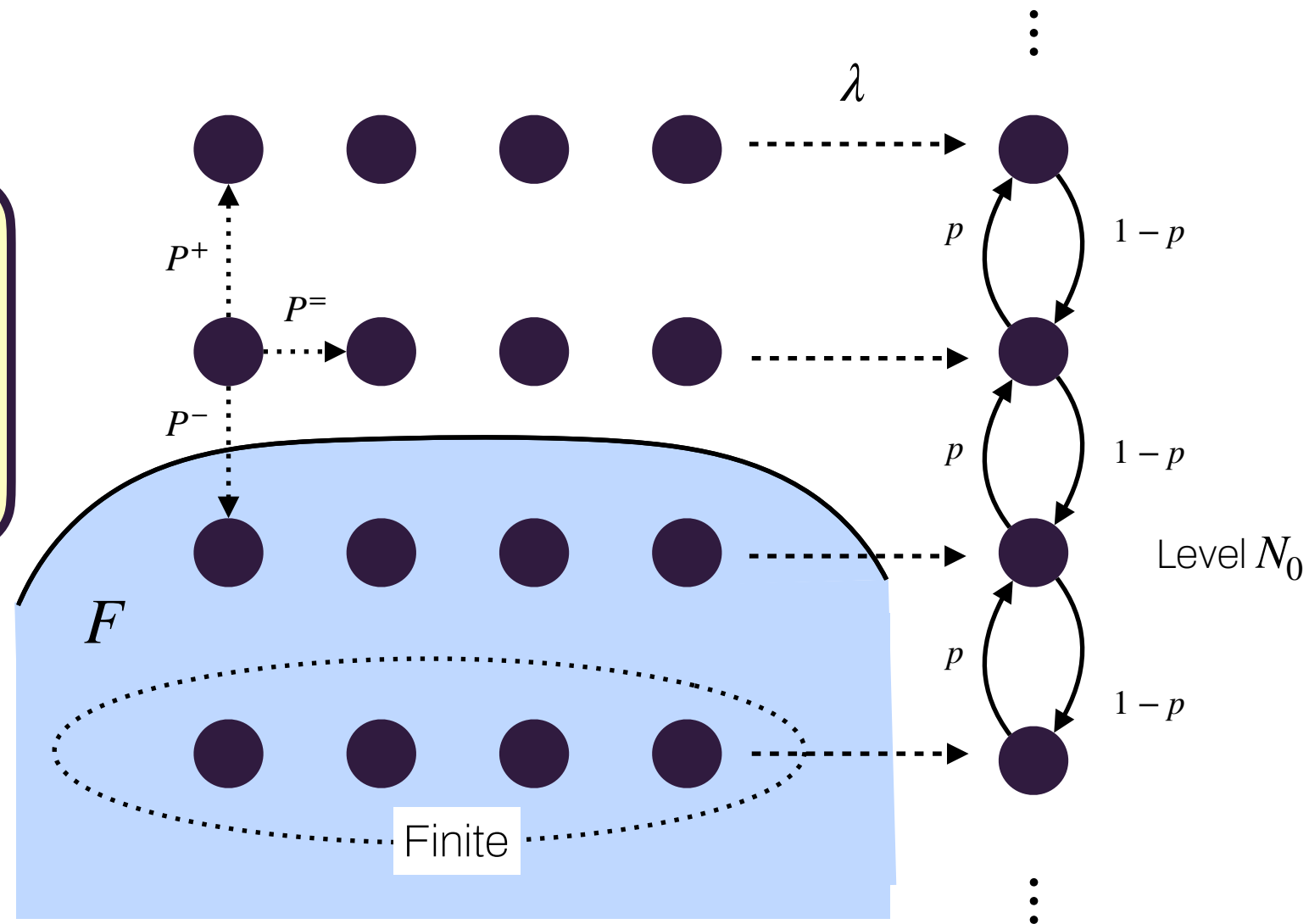
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Congestion
phenomenon



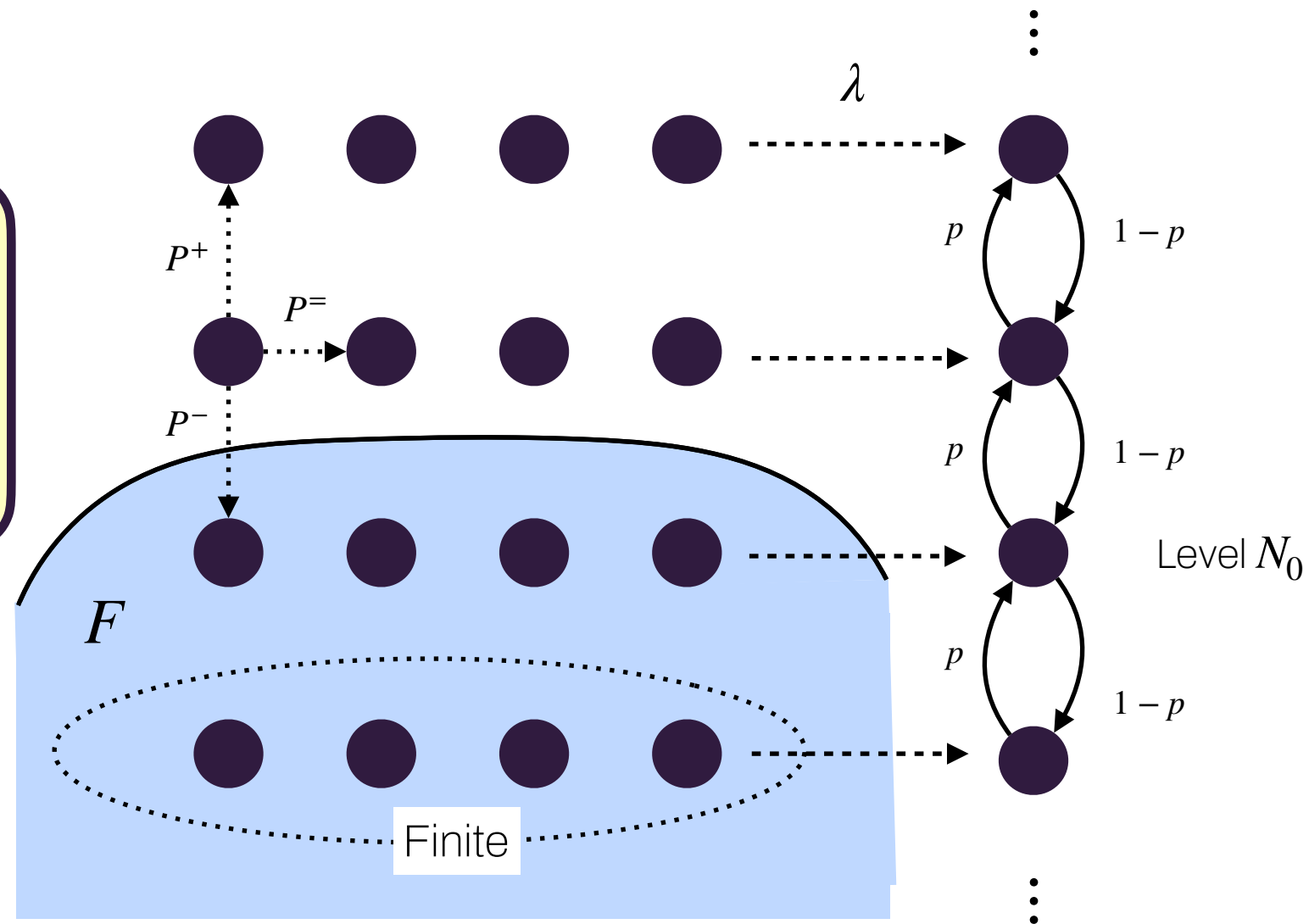
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Congestion phenomenon



$(\hat{p}, N_0)$ -divergence of $\mathcal{C}$ with $\frac{1}{2} < p < \hat{p}$ implies the random walk $\mathcal{C}_p^\bullet$ is an abstraction of $\mathcal{C}$

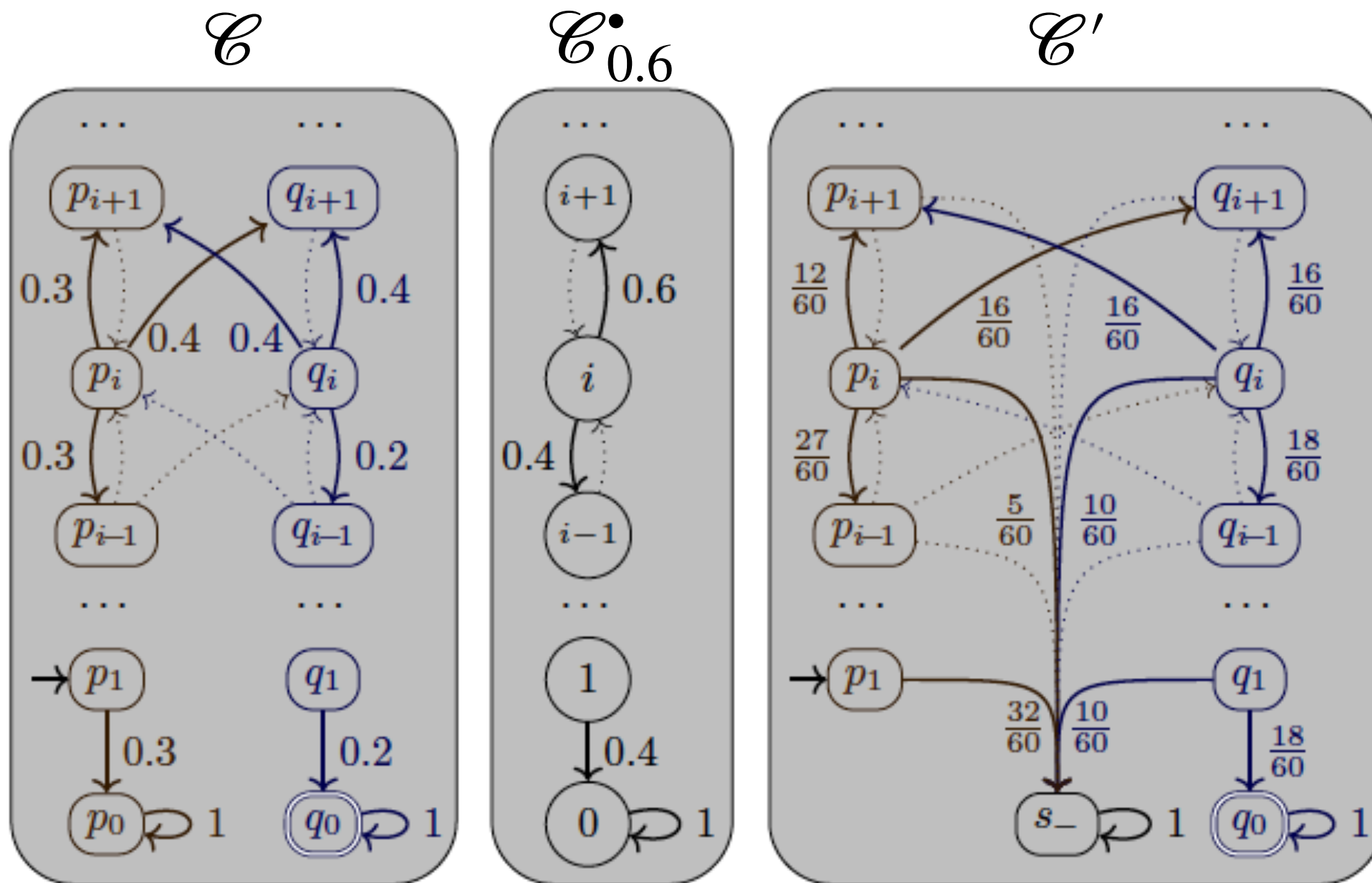
Correctness of the approach

Theorem

Let $\mathcal{C}$ be an LMC with level function λ , $\mathcal{C}_p^\bullet$ the random walk of parameter p . Assume there is N_0 and $\hat{p}$ s.t. $\frac{1}{2} < p < \hat{p}$ and $\mathcal{C}$ is $(\hat{p}, N_0)$ -divergent. Then:

- ▶ The analysis of $\mathcal{C}$ can be made via the biased Markov chain obtained by importance sampling through the abstraction $\mathcal{C}_p^\bullet$
- ▶ If furthermore, $\inf_{s \text{ s.t. } \lambda(s) > N_0} P^+(s) > 0$, then the expected time to sample an execution is finite

Example



$\mathcal{C}$ is not decisive

$\mathcal{C}'$ is decisive
+ finite sampling time

Implementation and experiments

<https://cosmos.lacl.fr/>

[BBDHP15] P. Ballarini, B. Barbot, M. Duflot, S. Haddad, N. Pekergin. Hasl: A new approach for performance evaluation and model checking from concepts to experimentation (Performance Evaluation)

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Note: in all experiments, the confidence is set to 99 %

First example

- ▶ State-free proba. pushdown automaton $\mathcal{C}: \{A \xrightarrow{1} C; A \xrightarrow{n} BB; B \xrightarrow{5} \varepsilon; B \xrightarrow{n} AA; C \xrightarrow{1} C\}$
- ▶ Start from A , and target the empty stack

First example

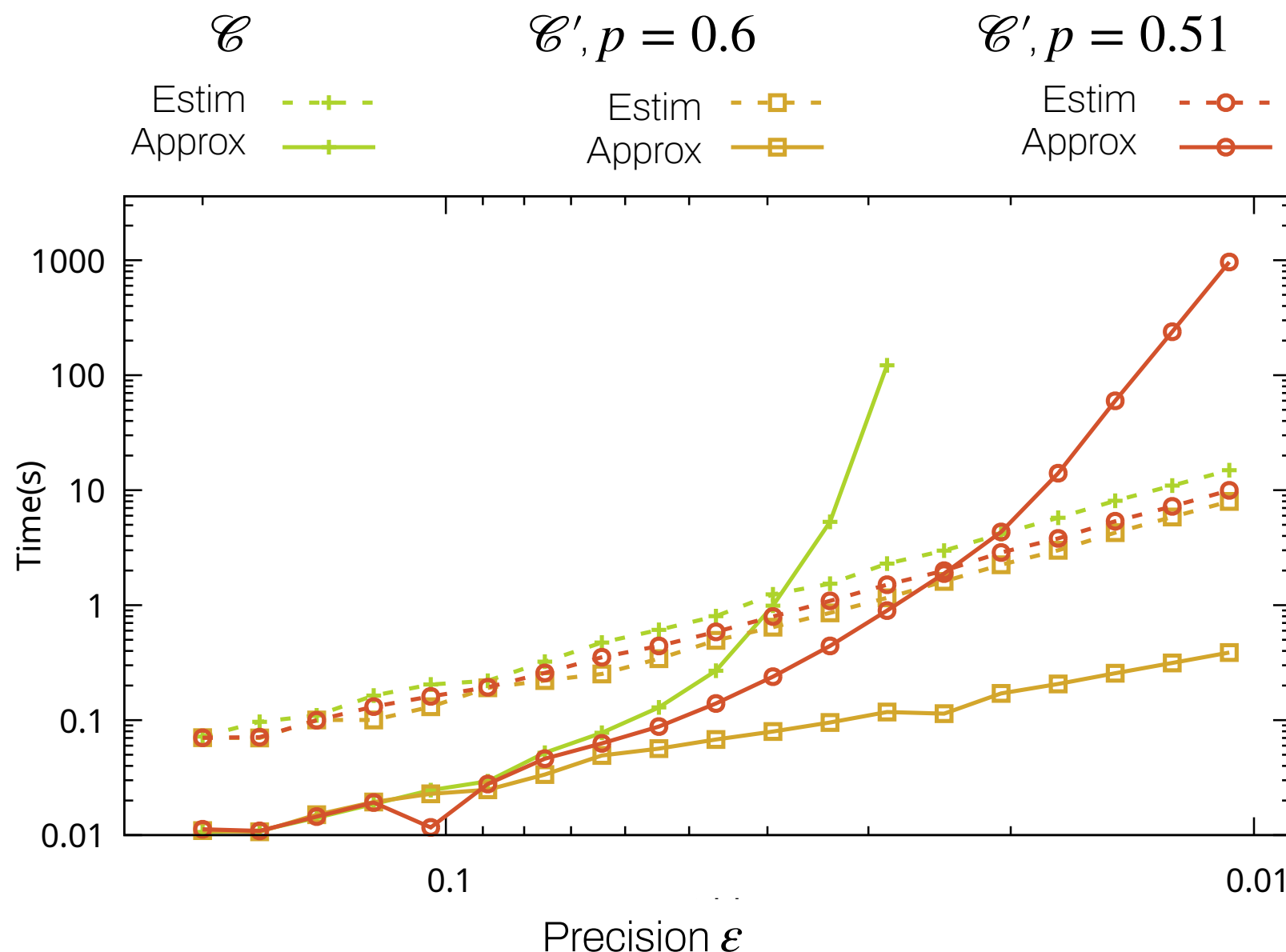
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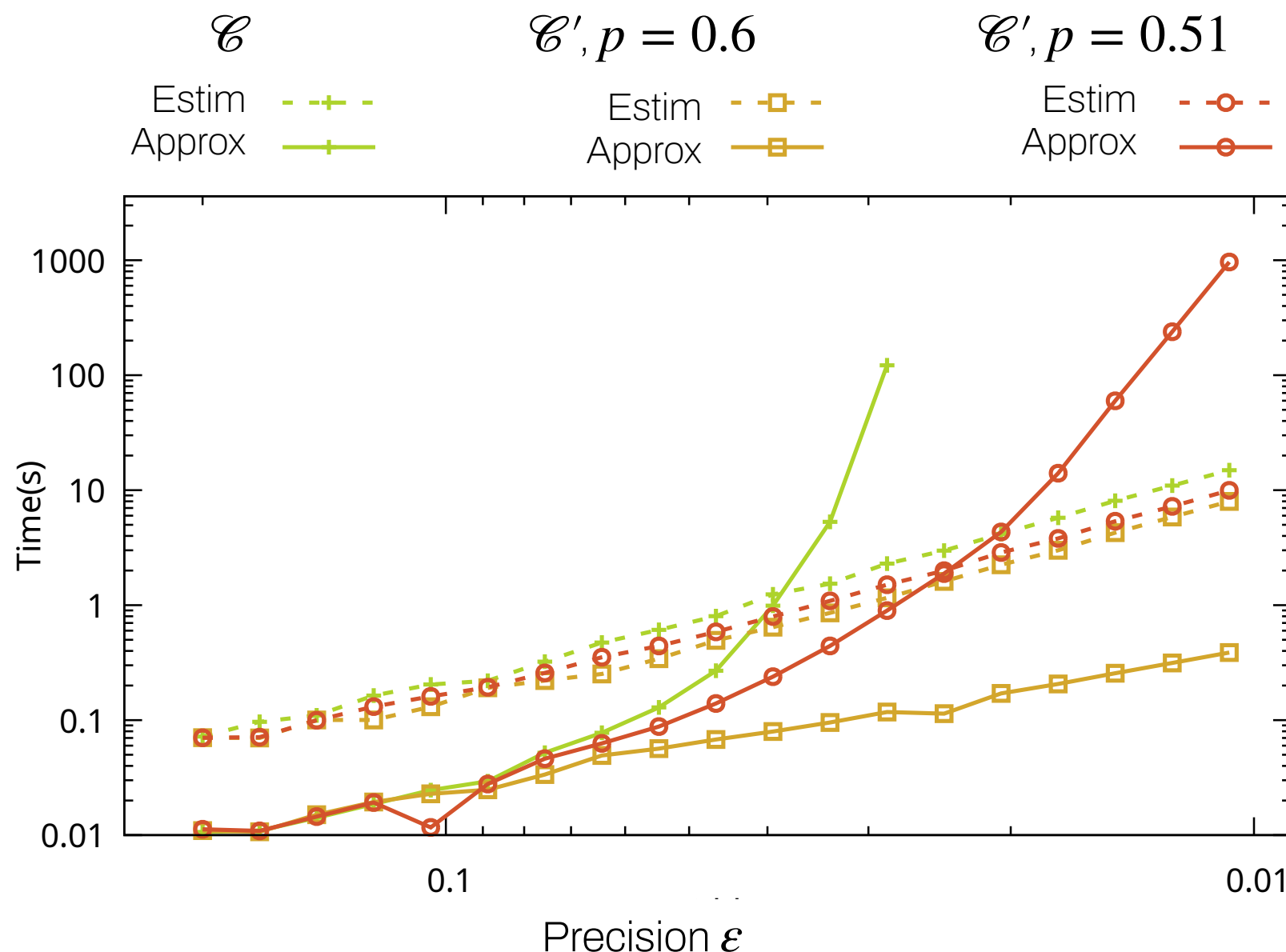
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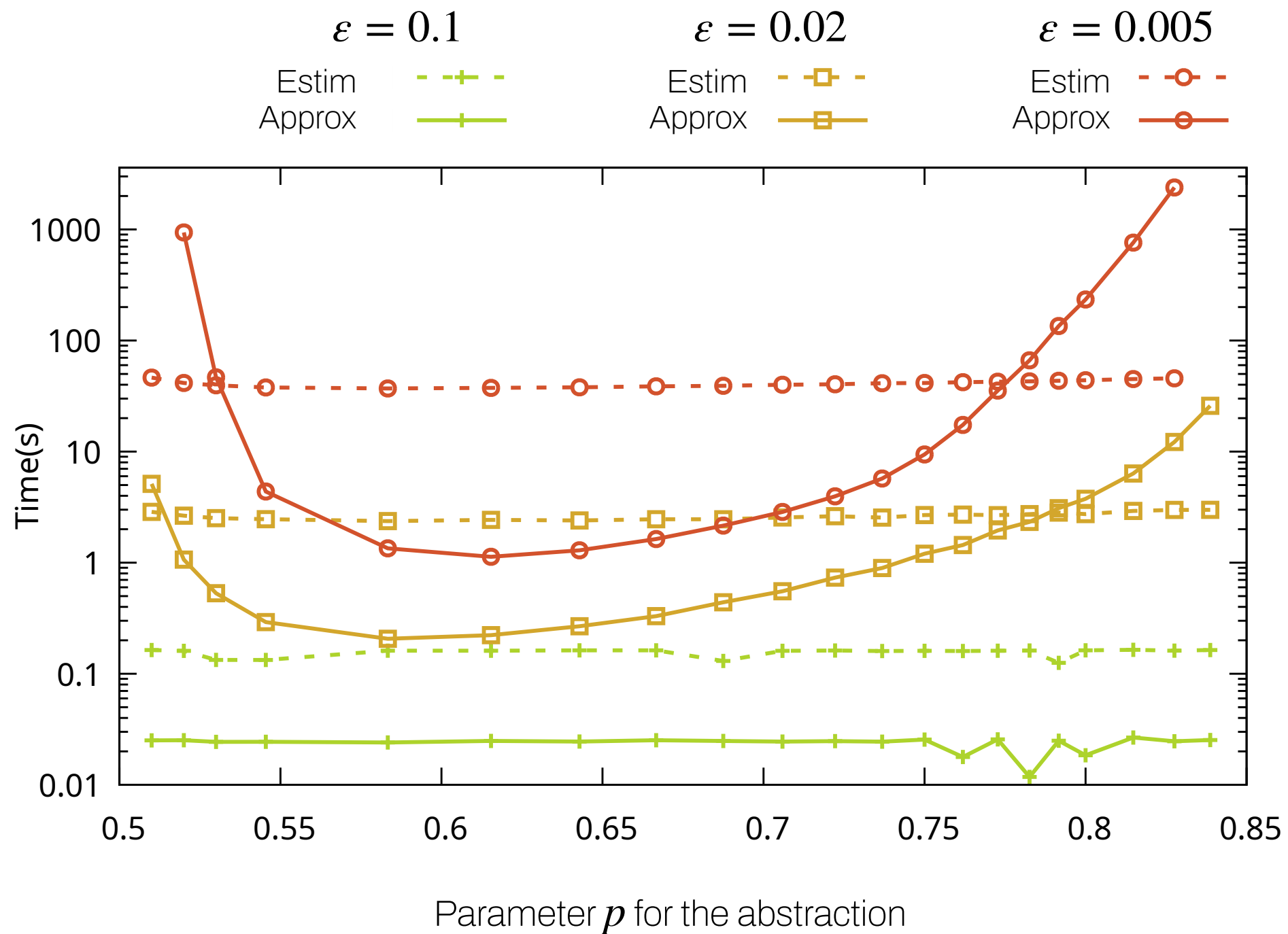
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- ▶ In Estim (SMC): doubling the precision impacts in square on computation time (slope 2 in this log-log scale)
- ▶ Importance sampling seems to improve the analysis time, both for Approx and Estim (no formal guarantee, though)
- ▶ There seems to be « a best p » ($p = 0.6$ here)
- ▶ For that best p , Approx behaves very well!

First example — continued



Second example

- ▶ State-free proba. pushdown automaton $\mathcal{C}$:
 $\{A \xrightarrow{1} B; A \xrightarrow{1} C; B \xrightarrow{10} \varepsilon; B \xrightarrow{10+n} AA; C \xrightarrow{10} A; C \xrightarrow{10+n} BB\}$
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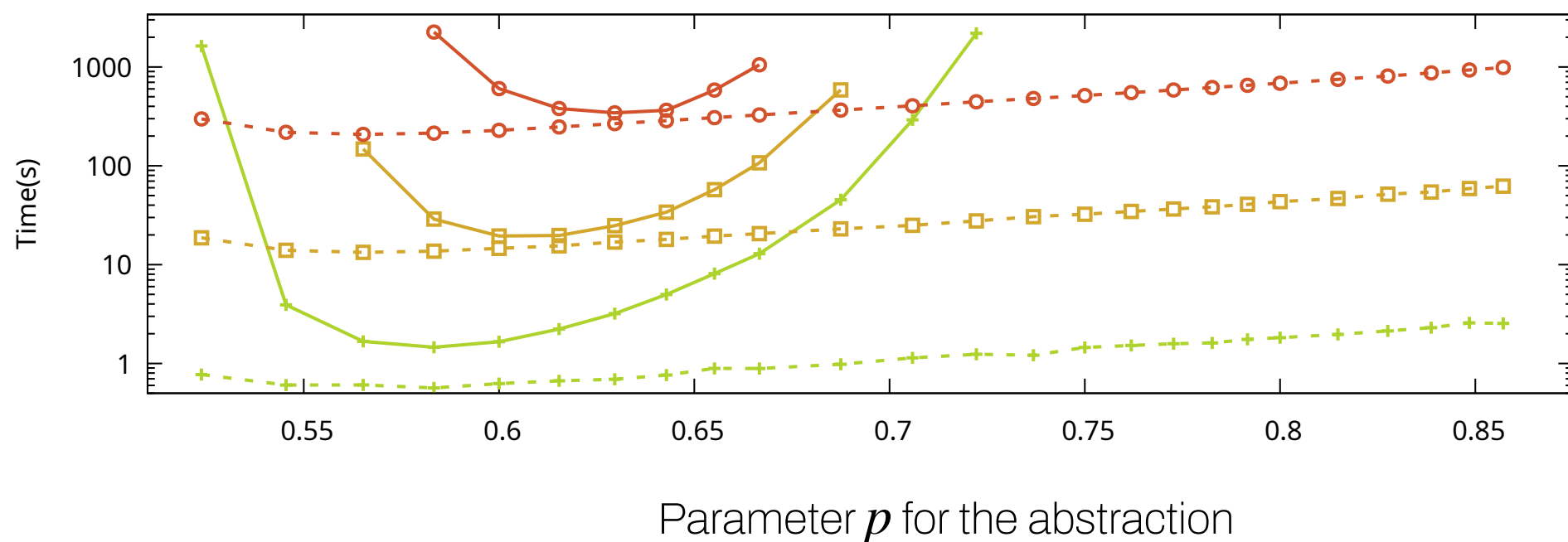
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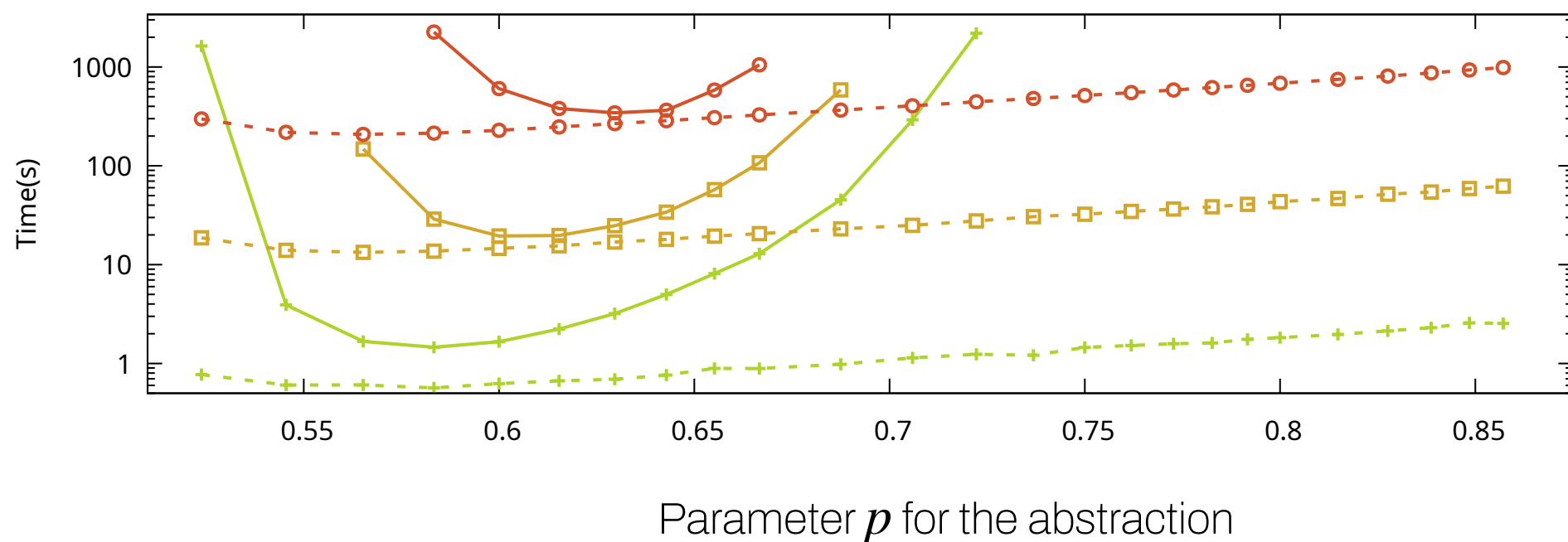


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- ▶ Estim (SMC) not too sensitive to p
 - Nevertheless (log scale): clear bell effect on p
- ▶ Approx very sensitive to p

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- p big: large desactivation zone (N_0)
- p small: small bias (few trajectories end up in 😞)