Beyond Decisiveness of Infinite Markov Chains

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Purpose of this work

Design algorithms to estimate probabilities in some **infinite-state** Markov chains, **with guarantees**

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Our contributions

- Review two existing approaches (approximation algorithm and estimation algorithm) and specify the required hypothesis for correctness
- Propose an approach based on **importance sampling** and **abstraction** to partly relax the hypothesis
- Analyze empirically the approaches

Discrete-time Markov chains

Discrete-time Markov chain (DTMC)

$\mathscr{C} = (S, s_0, \delta)$ with S at most denumerable, $s_0 \in S$ and $\delta : S \to \text{Dist}(S)$

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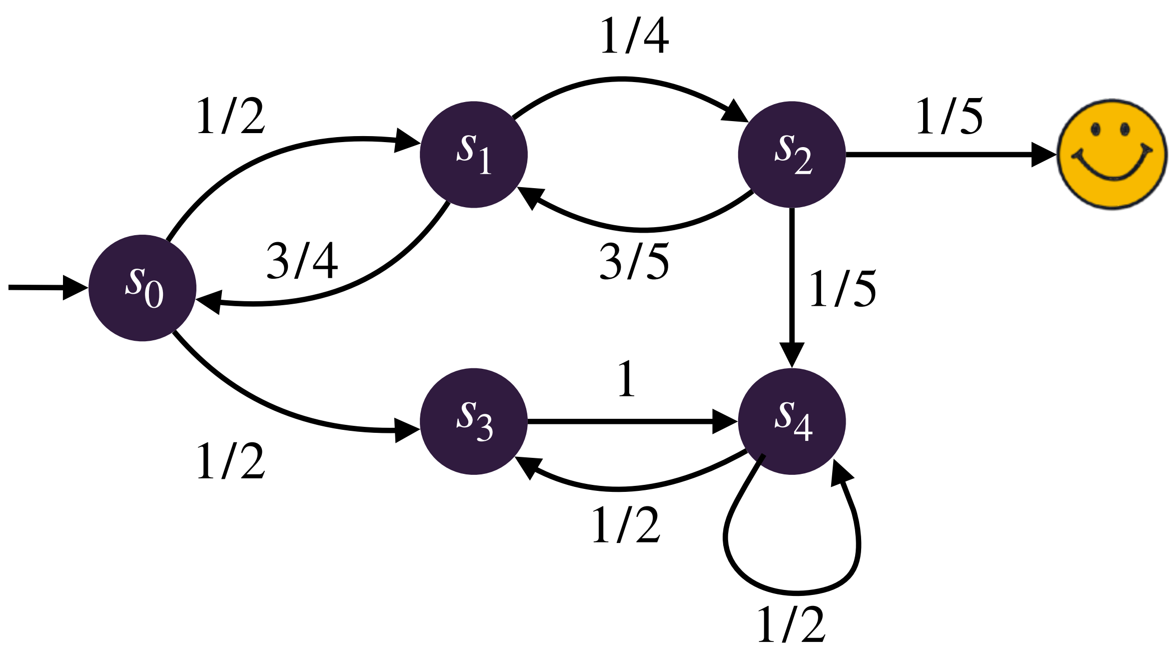

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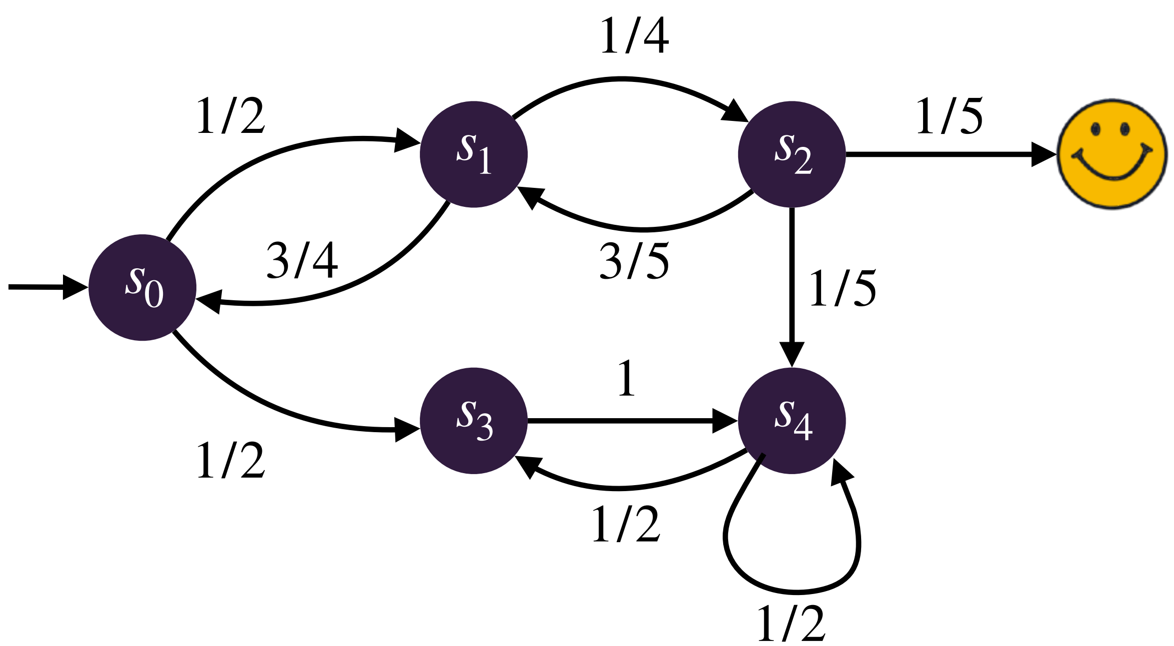

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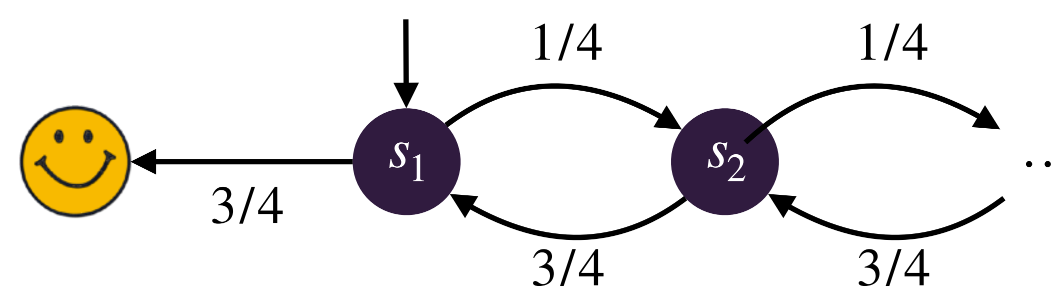

Denumerable Markov chain
(random walk of parameter $1/4$)

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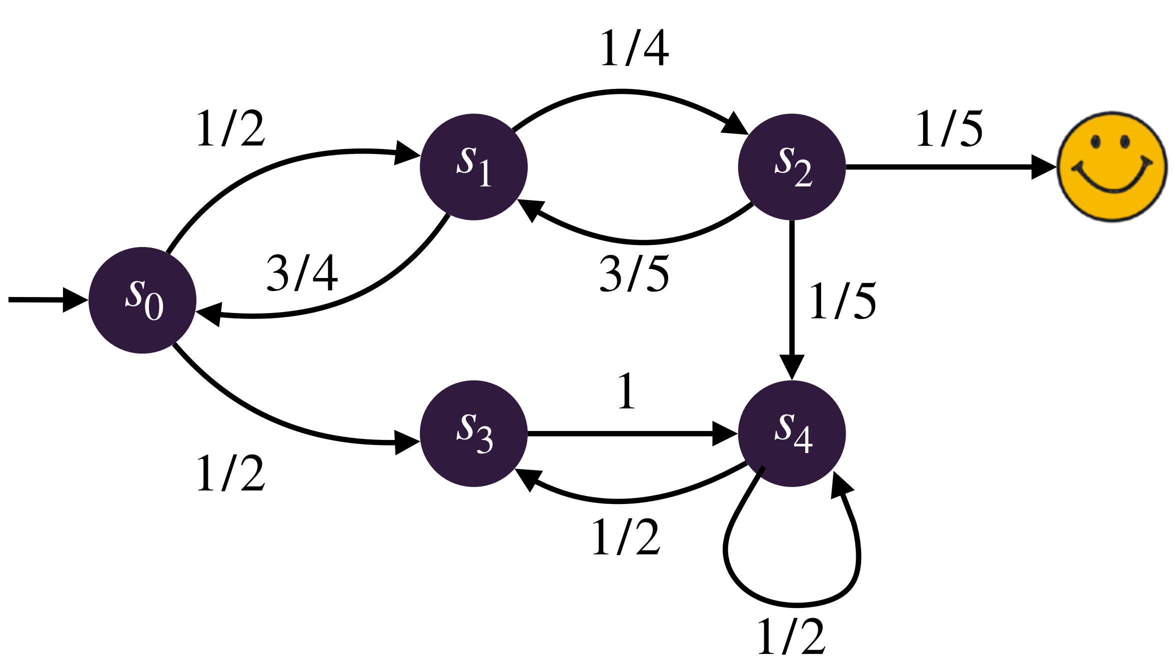

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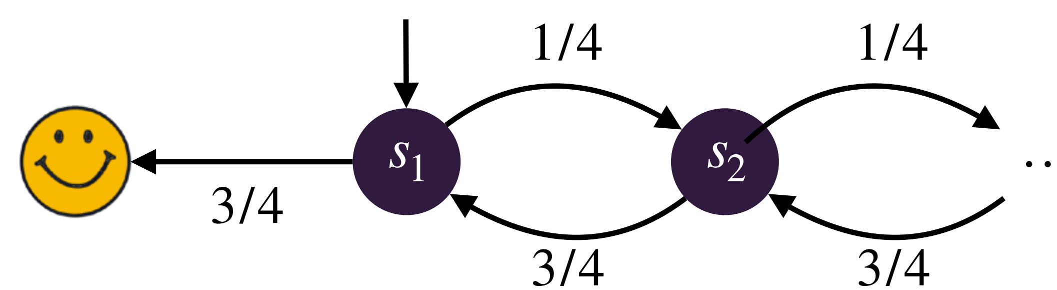

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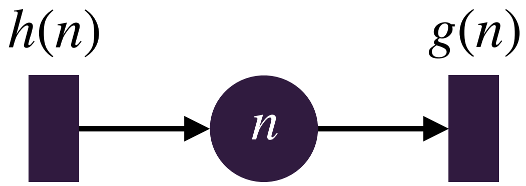
High-level models for (infinite) Markov chains

- Queues



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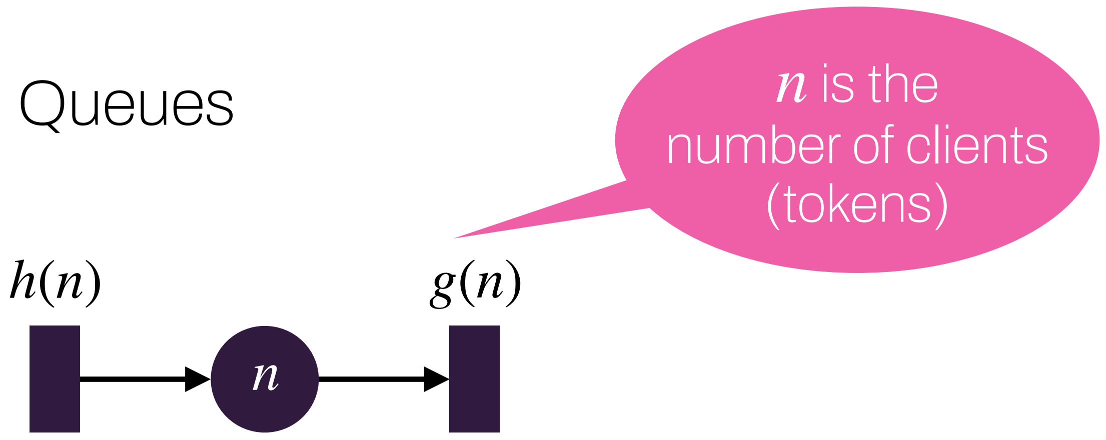

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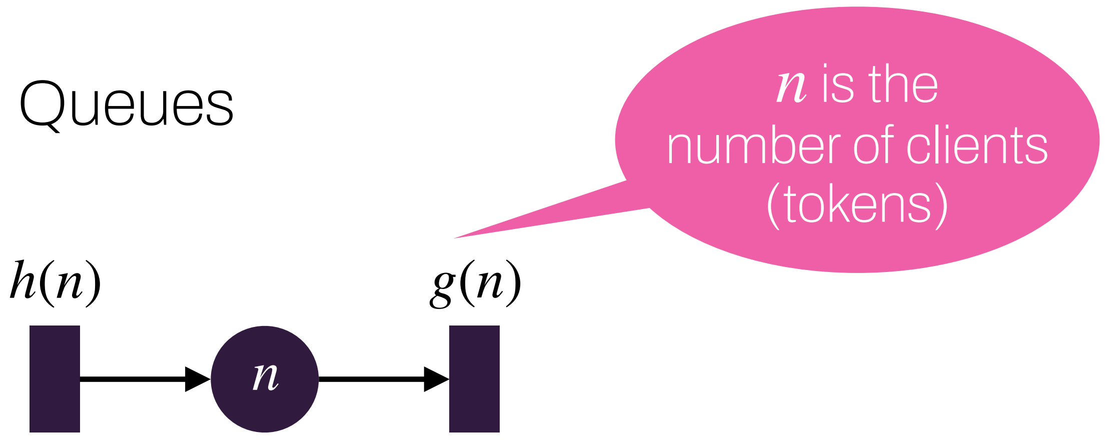

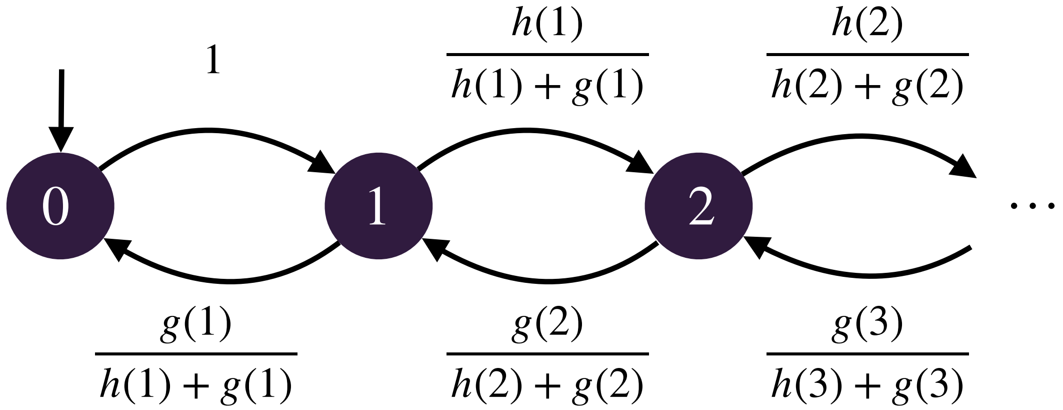

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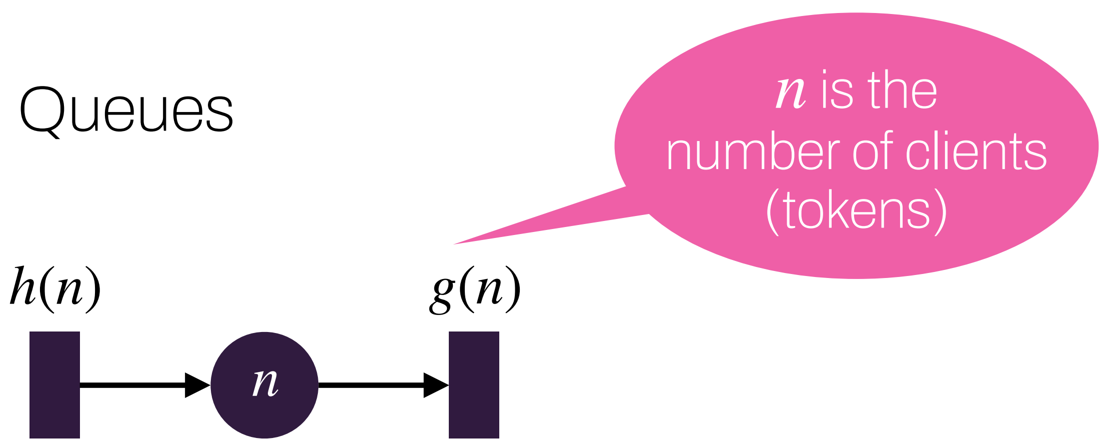

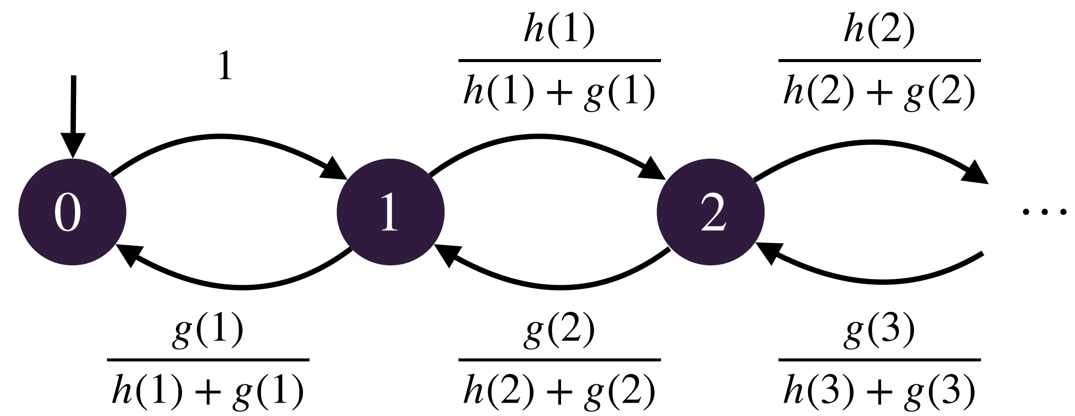

- Probabilistic pushdown automata

$$A \xrightarrow{1} C \quad A \xrightarrow{n} BB \quad B \xrightarrow{5} \varepsilon$$
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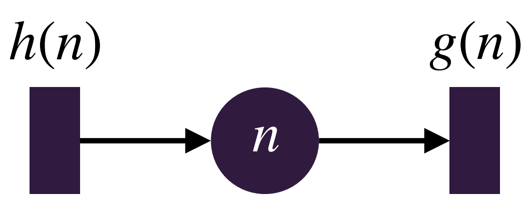

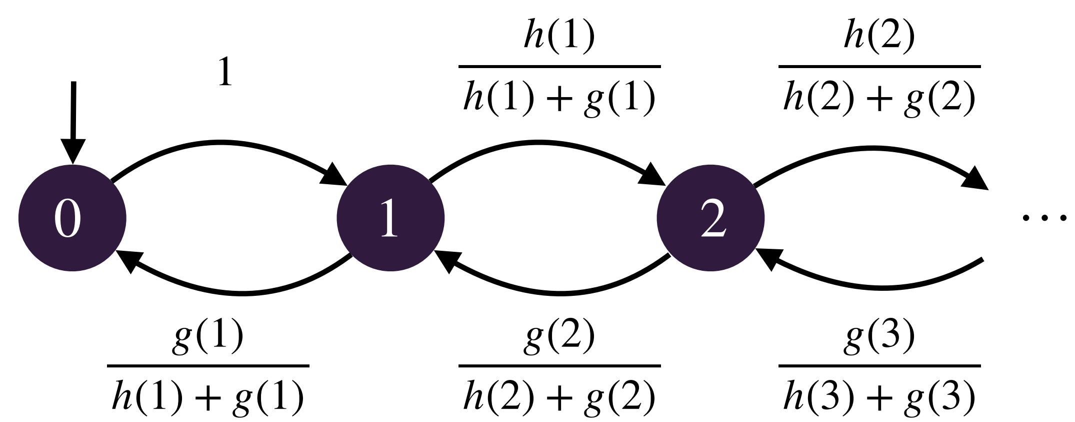

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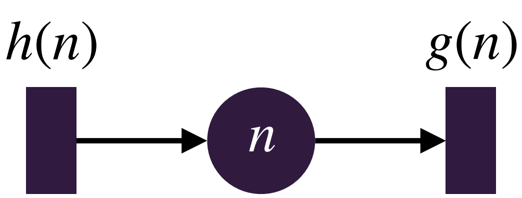

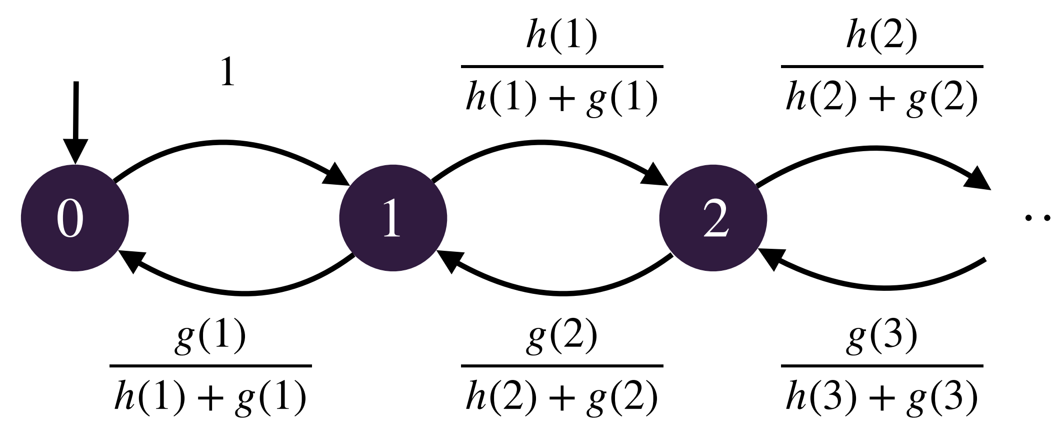

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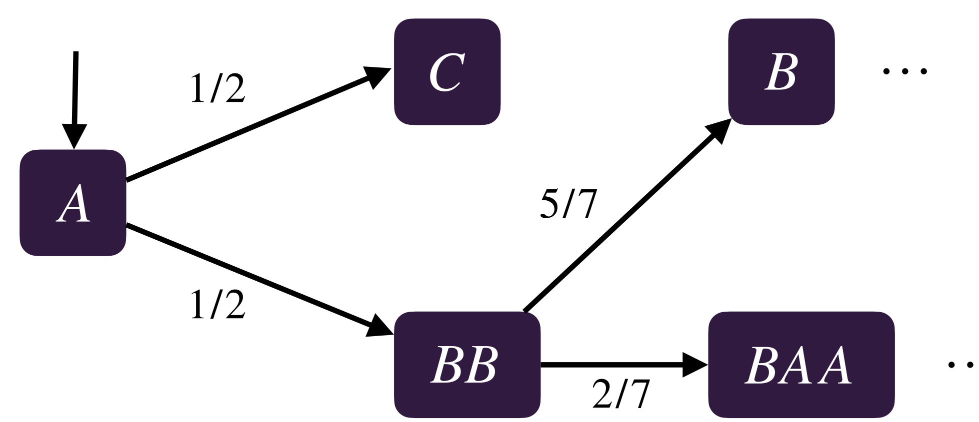
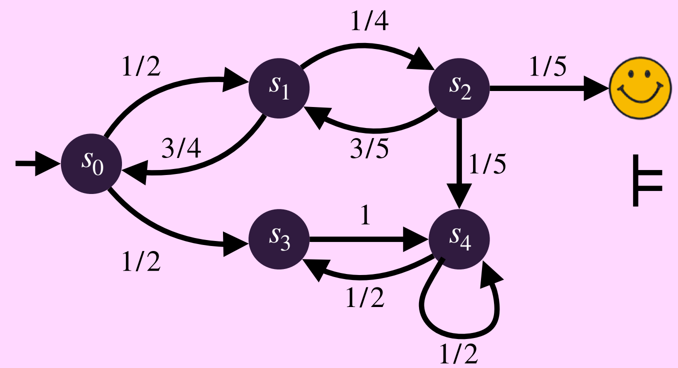


Quantitative analysis of Markov chains

Compute $\mathbb{P}\Big($ 

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[RKNP04] J. Rutten, M. Kwiatkowska, G. Norman, D. Parker. Mathematical techniques for analyzing concurrent and probabilistic systems (Monograph)

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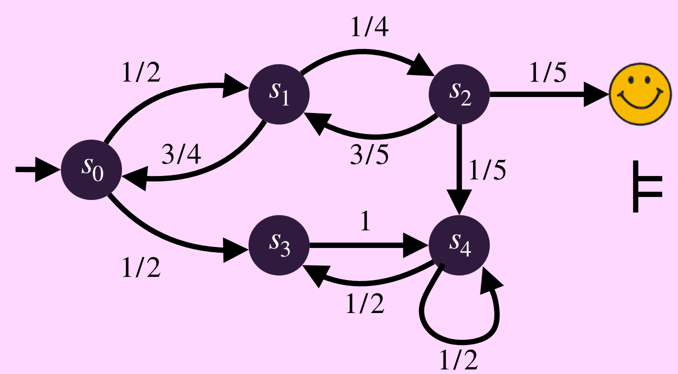

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Closed-form solution

- Random walk of parameter $p > 1/2$:
  $\mathbb{P}_{s_n}(\mathbf{F}$ 😊 $) = \kappa^n$, where $\kappa = \dfrac{1-p}{p}$
- Does not always exist

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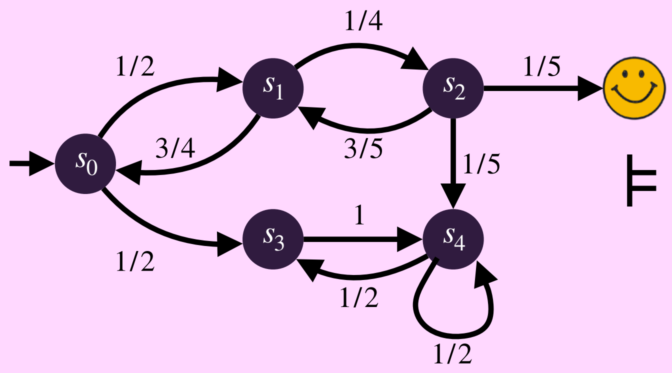
Compute $\mathbb{P}\Big($ 

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- $x_s = \begin{cases} 1 & \text{if } s = \text{🙂} \\ 0 & \text{if } s \not\models \exists \mathbf{F}\text{🙂} \\ \sum_t \mathbb{P}(s \to t) \cdot x_t & \text{otherwise} \end{cases}$
- $\mathbb{P}_{s_0}(\mathbf{F}$ 🙂 $) = 1/19$
- System must be finite
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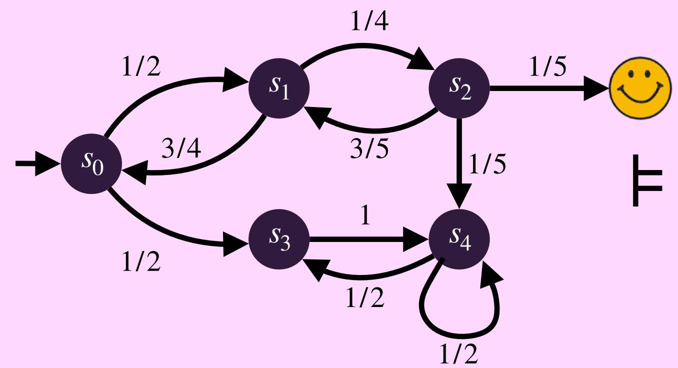
Compute $\mathbb{P}\Big($  $\models \mathbf{F}$ ☺ $\Big)$

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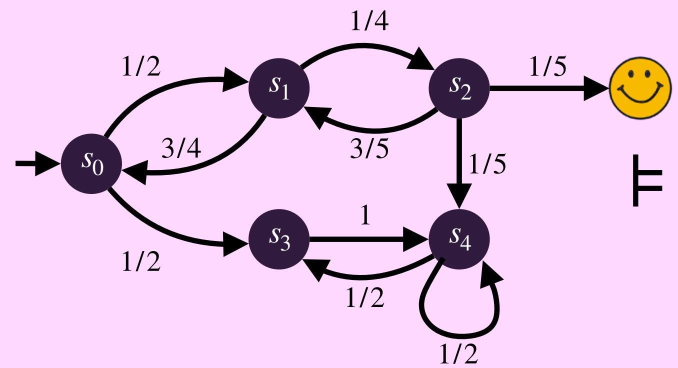
Compute $\mathbb{P}\Big($ 

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- Specific approaches for **decisive** Markov chains

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Decisiveness

☹️ $= \{s \in S \mid s \not\models \exists \mathbf{F}$ 🙂 $\}$

Decisiveness

A DTMC $\mathscr{C}$ is decisive from s w.r.t. 🙂 if $\mathbb{P}_s(\mathbf{F}$ 🙂 $\vee\, \mathbf{F}$ ☹️ $) = 1$

[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)

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- Examples of decisive Markov chains: finite Markov chains, probabilistic lossy channel systems, probabilistic VASS, noisy Turing machines, ...

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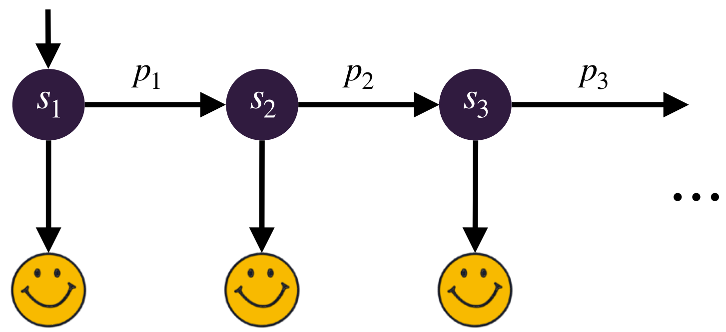

- $\mathbb{P}(\mathbf{G}\neg 🙂) = \prod_{i \geq 1} p_i$
- Decisive iff this product equals 0

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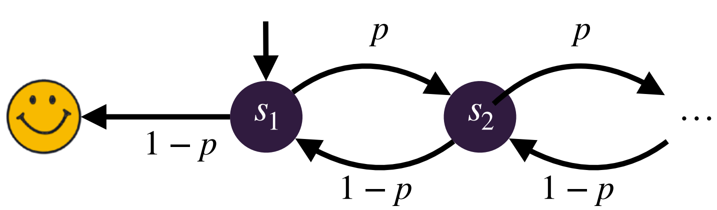

- Recurrent random walk ($p \leq 1/2$): decisive
- Transient random walk ($p > 1/2$): not decisive

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Approximation scheme

- Aim: compute probability of $\mathbf{F}$ 
-  $= \{ s \in S \mid s \not\models \exists \mathbf{F}$  $\}$

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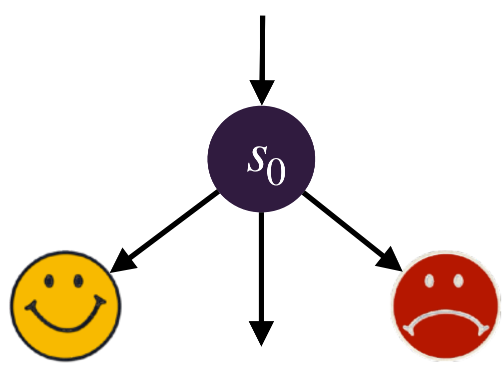

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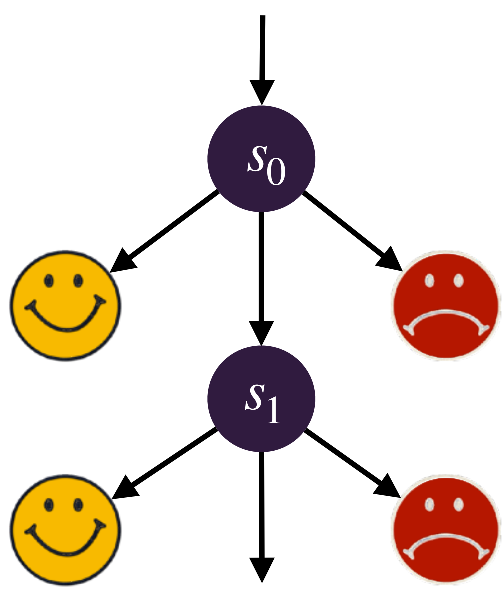

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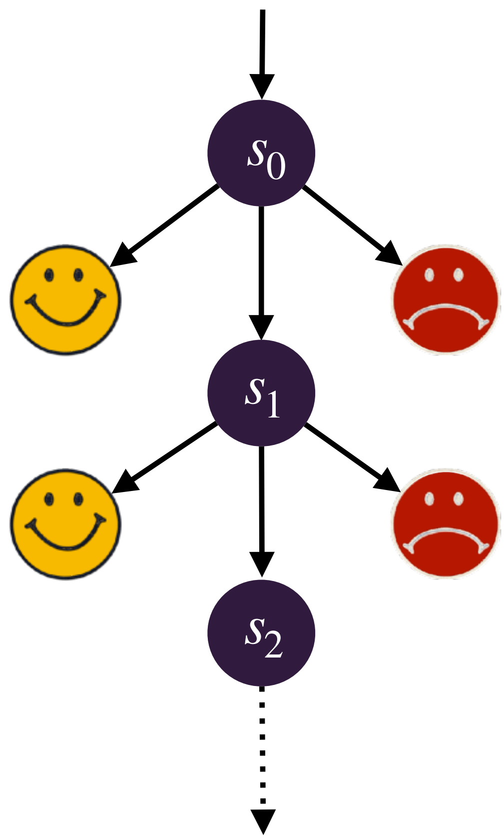

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$$\text{IA} \qquad\qquad \text{VI}$$

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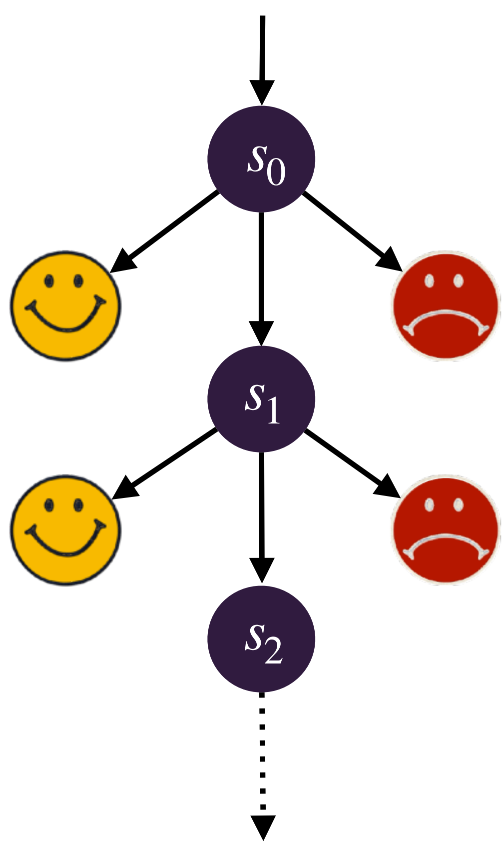

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At the limit: $\mathbb{P}(\mathbf{F}$ 🙂$)$ $\qquad 1 - \mathbb{P}(\mathbf{F}$ ☹️$)$

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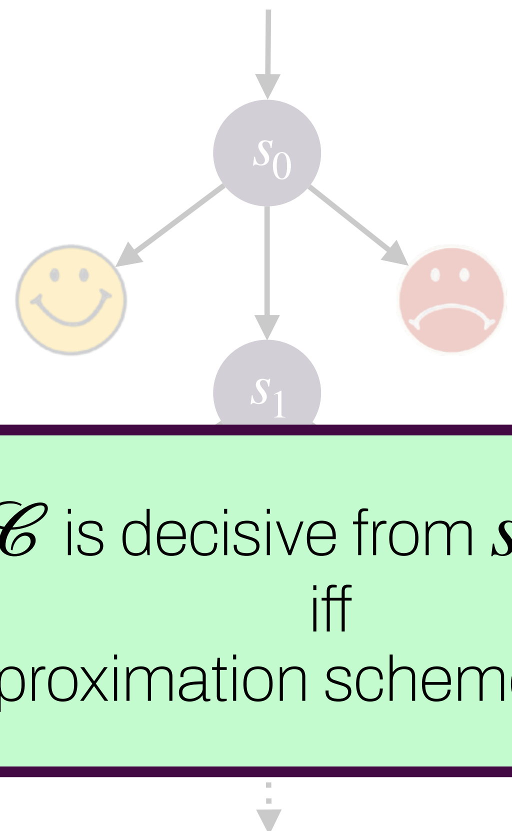

$\mathscr{C}$ is decisive from s_0 w.r.t. 🙂
iff
the approximation scheme converges

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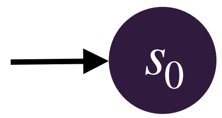
Statistical model-checking

Sample N paths



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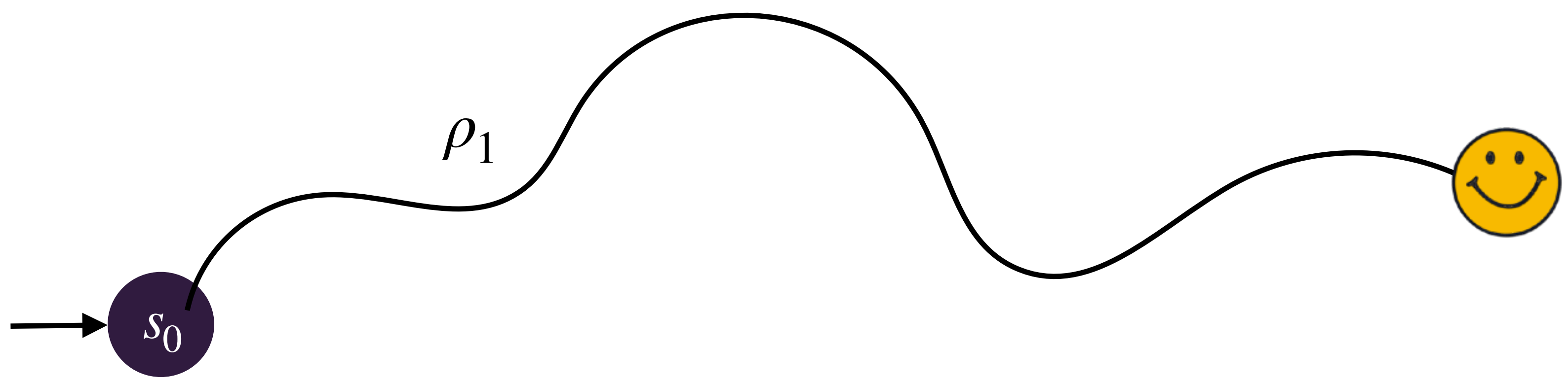

$n_1 = 1$

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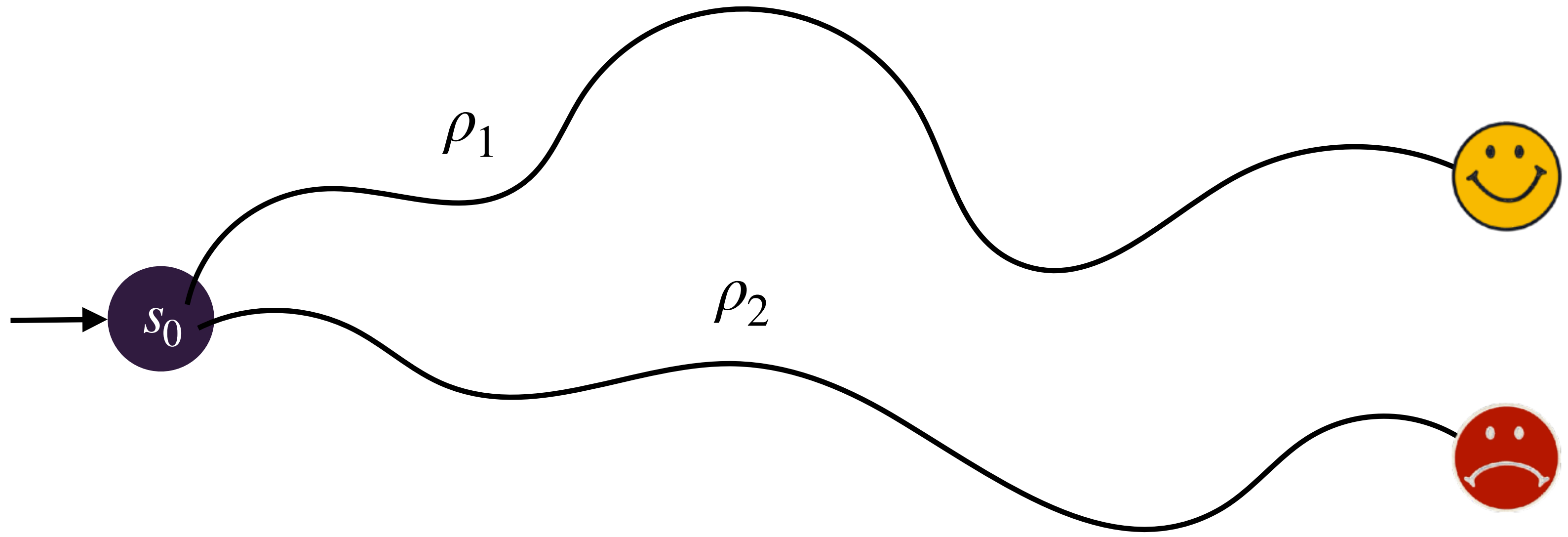

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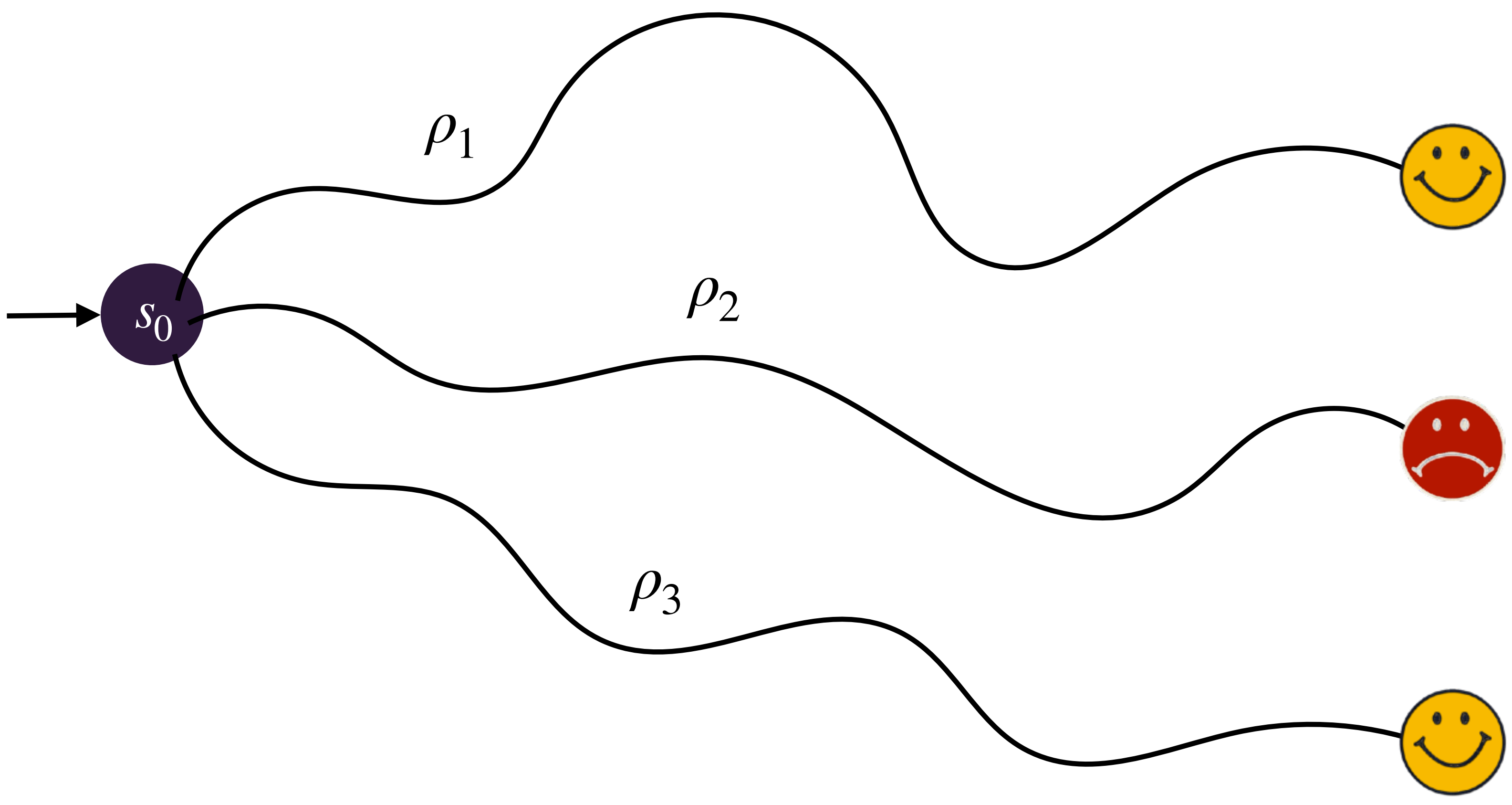


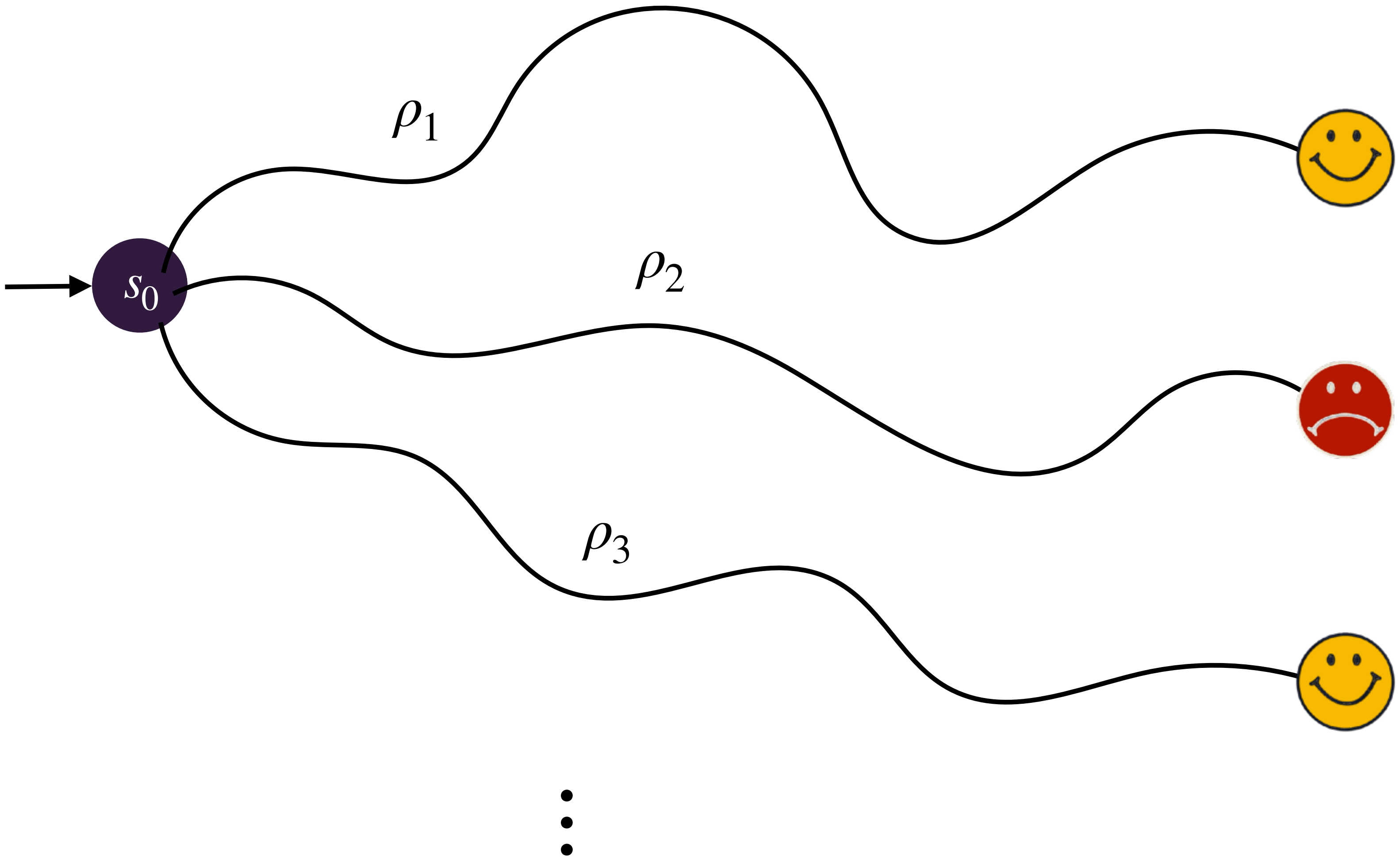
$n_1 = 1$

$n_2 = n_1$

$n_3 = n_2 + 1$

Statistical model-checking

Sample N paths



$n_1 = 1$

$n_2 = n_1$

$n_3 = n_2 + 1$

⋮

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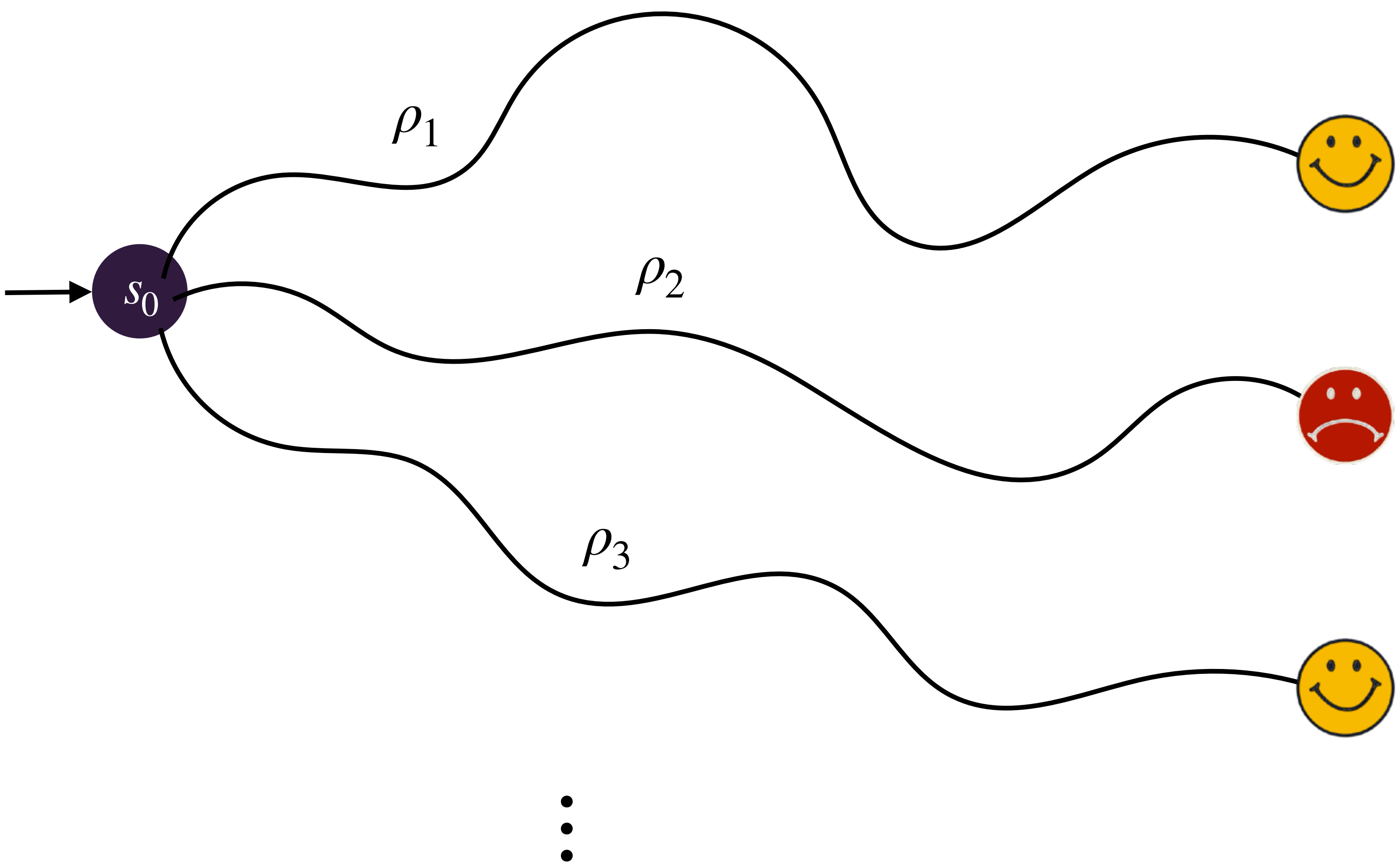

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$\vdots$

Return $\dfrac{n_N}{N}$ + some confidence interval

Termination, efficiency and guarantees

Termination (To our knowledge, never expressed like this)

$\mathscr{C}$ is decisive from s_0 w.r.t. 
iff
a sampled path starting at s_0 almost-surely hits  or 

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Guarantees: Hoeffding's inequalities

Let $\varepsilon, \delta > 0$, let $N \geq \frac{8}{\varepsilon^2} \log\left(\frac{2}{\delta}\right)$. Then:

$$\mathbb{P}\left(\left| \frac{n_N}{N} - \mathbb{P}(\mathbf{F}\ 🙂\ ) \right| \geq \frac{\varepsilon}{2} \right) \leq \delta$$

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Empirical average

Confidence value

Precision

$\left[\frac{n_N}{N} - \frac{\varepsilon}{2}; \frac{n_N}{N} + \frac{\varepsilon}{2}\right]$: confidence interval

What can we do for
non-decisive Markov chains??

Importance sampling [KH51]

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[BHP12] B. Barbot, S. Haddad, C. Picaronny. Coupling and Importance Sampling for Statistical Model Checking (TACAS'12)

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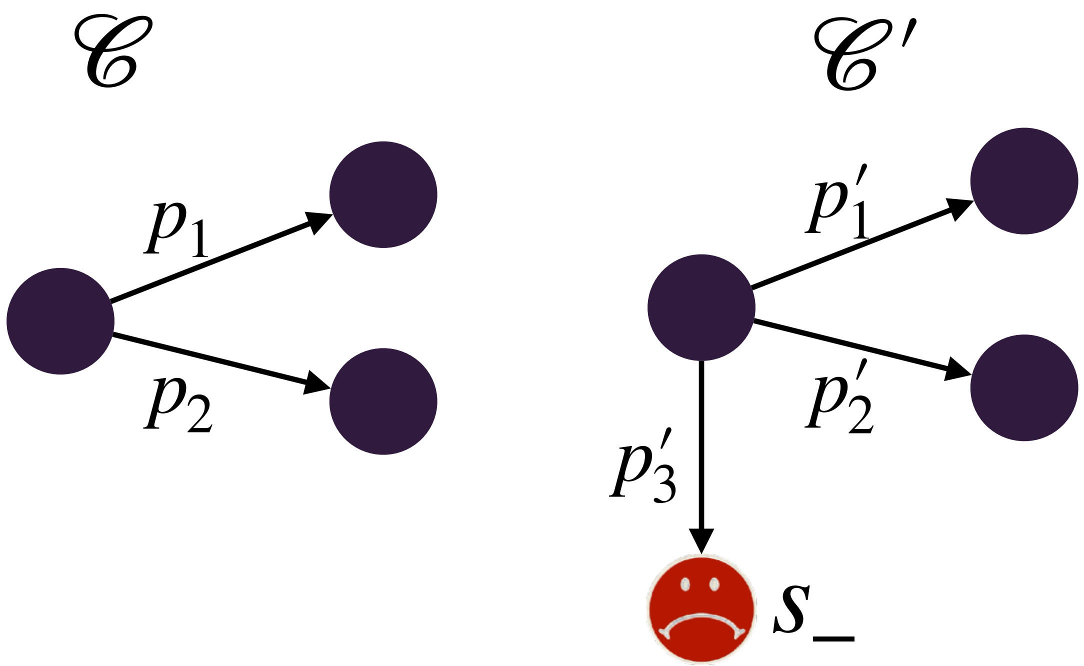

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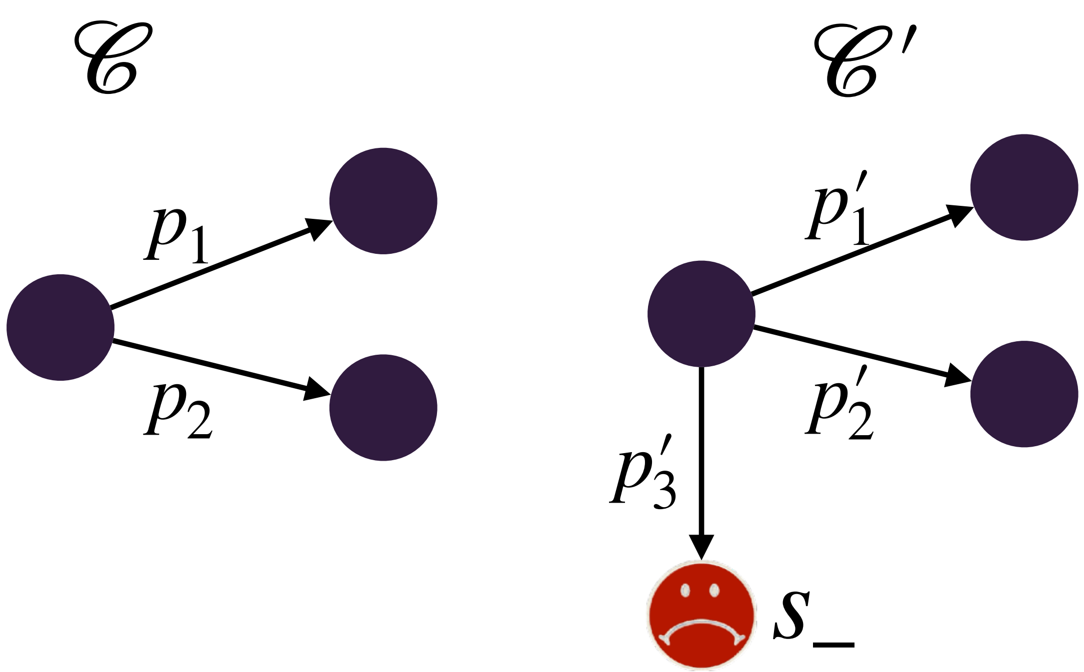

Correct the bias

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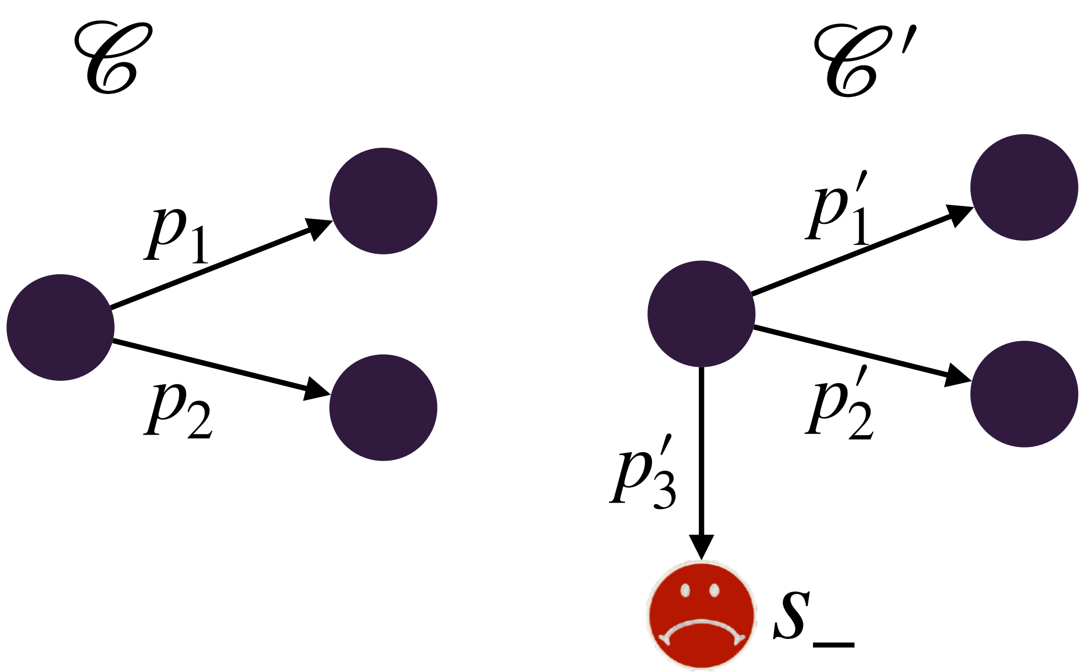

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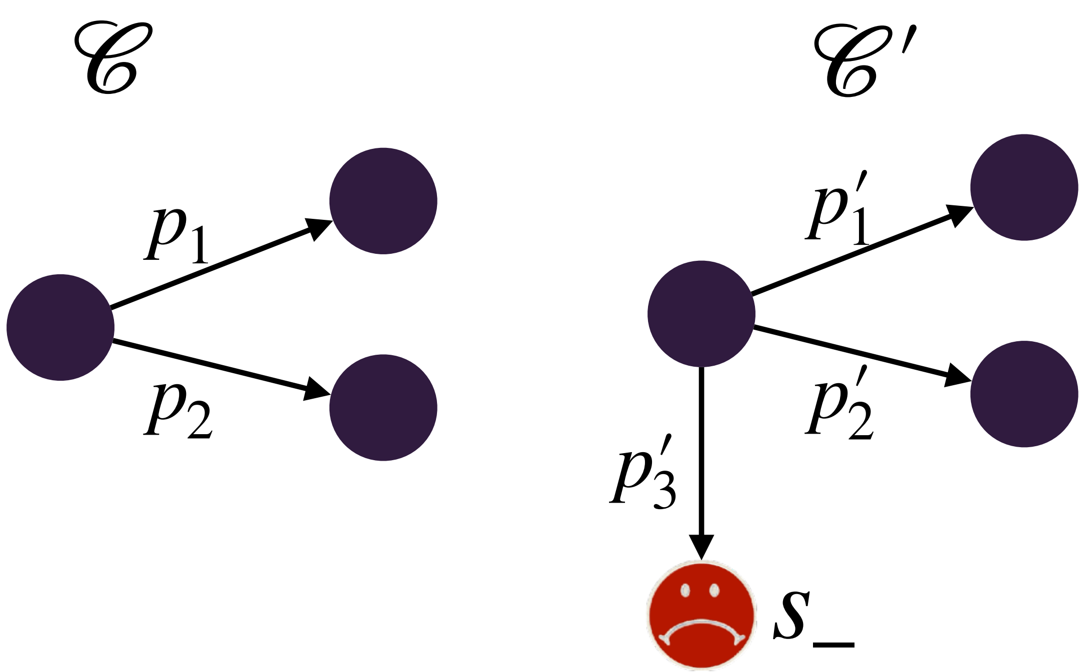

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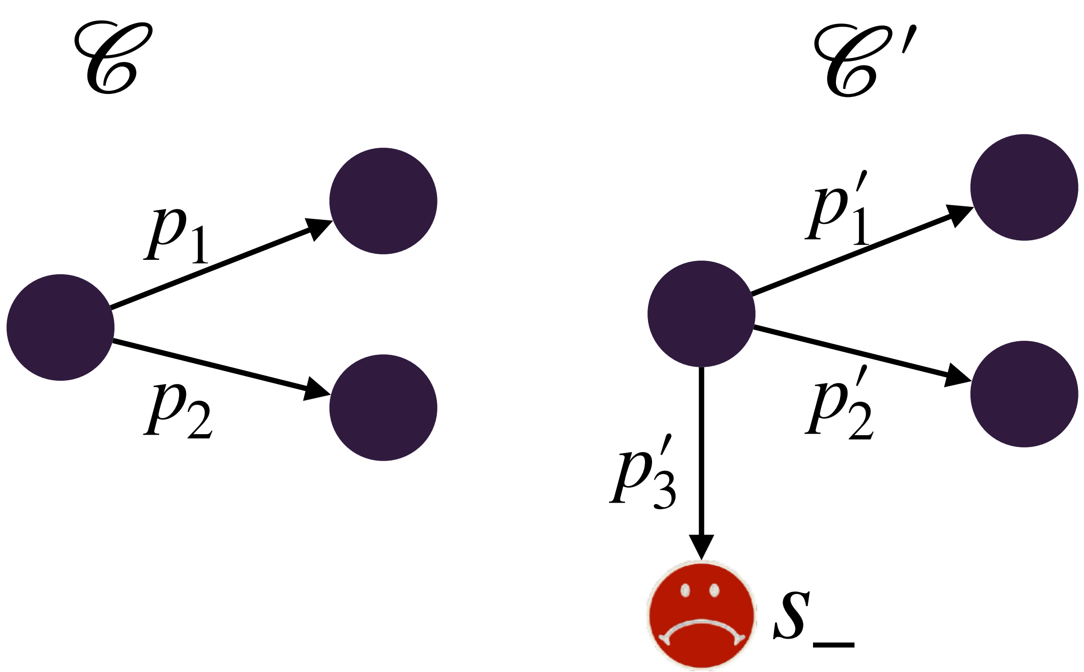

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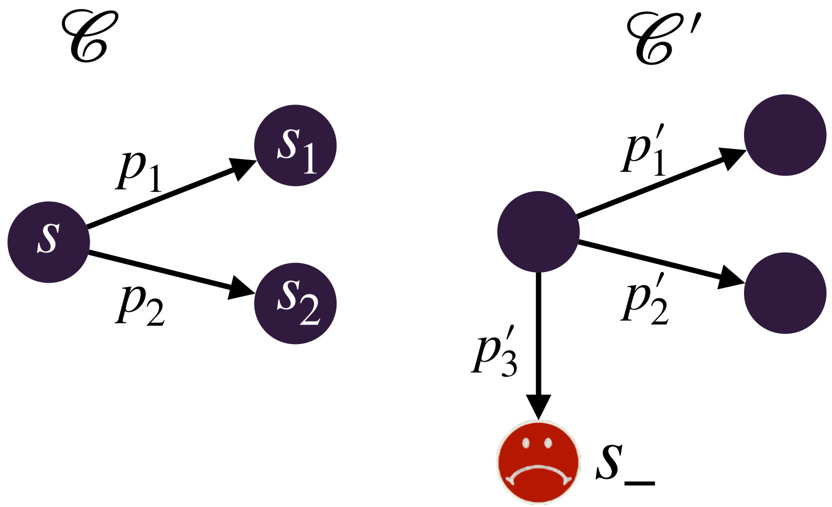

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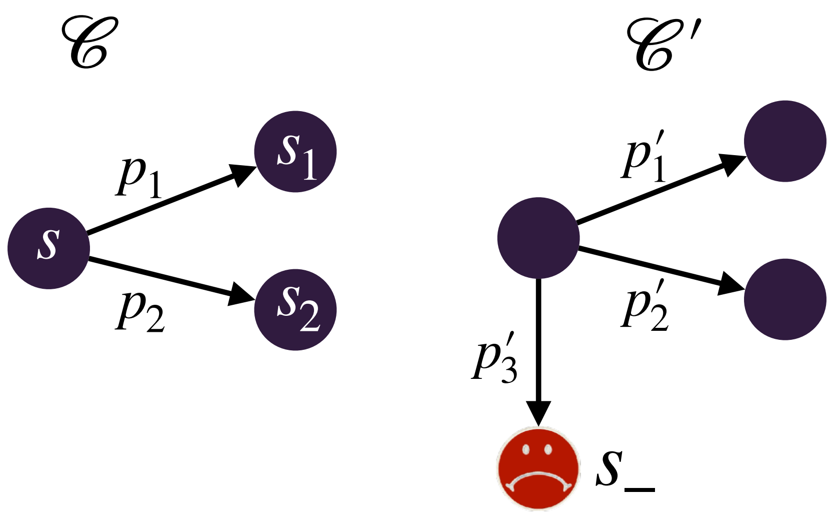

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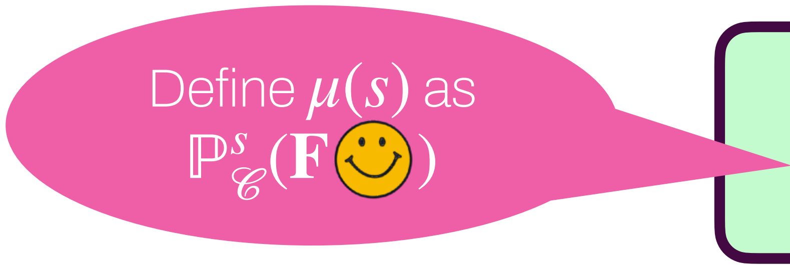

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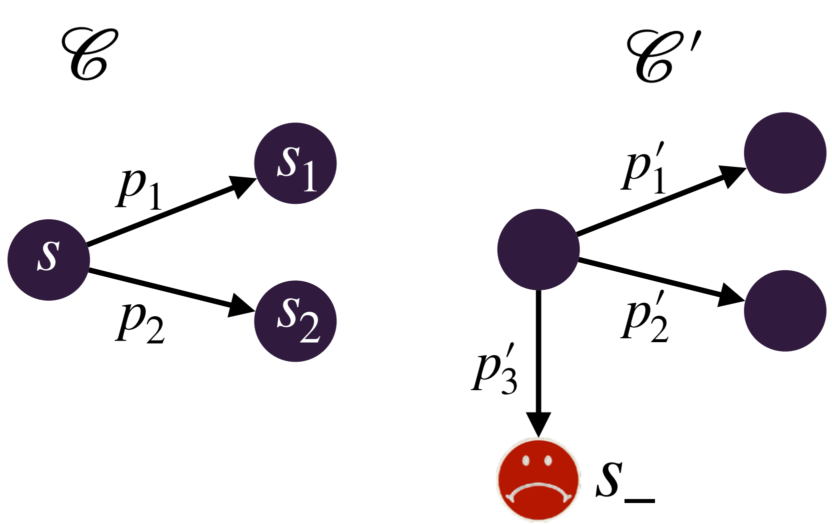

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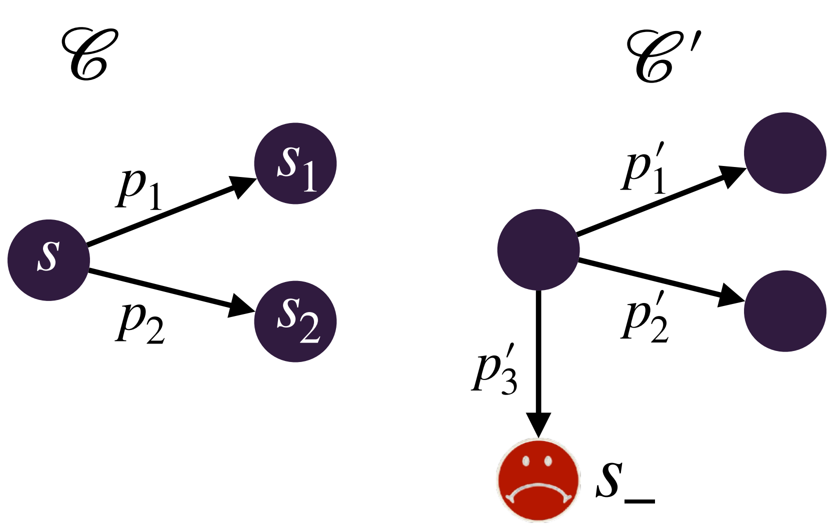

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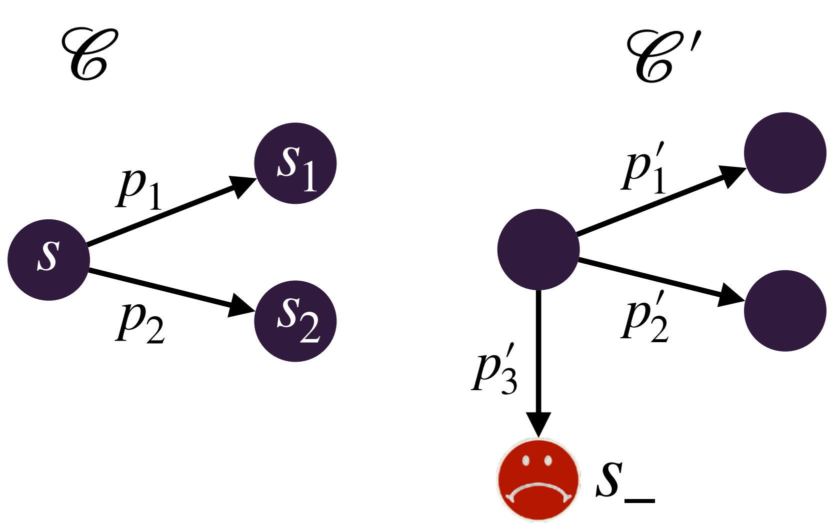

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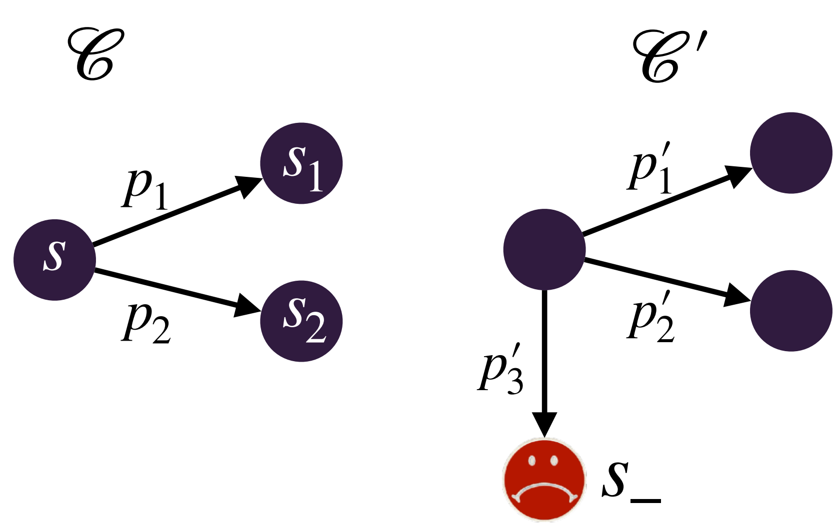

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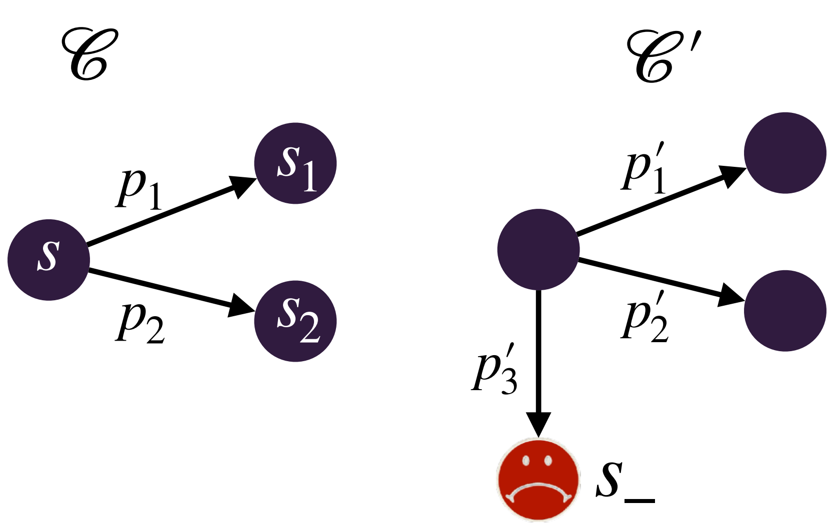

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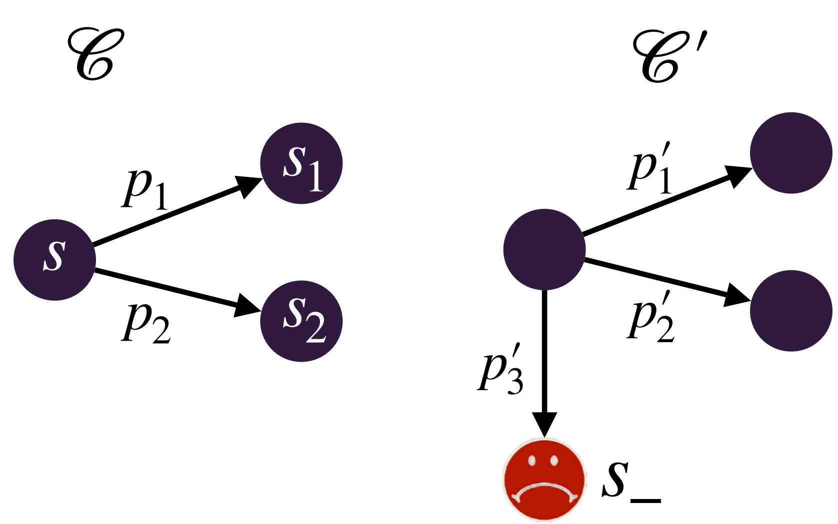
There is a best choice: $p'_i = \dfrac{\mu(s_i)}{\mu(s)} \cdot p_i$

- The r.v. in $\mathscr{C}'$ takes value $\mu(s)$
- One needs to know μ!



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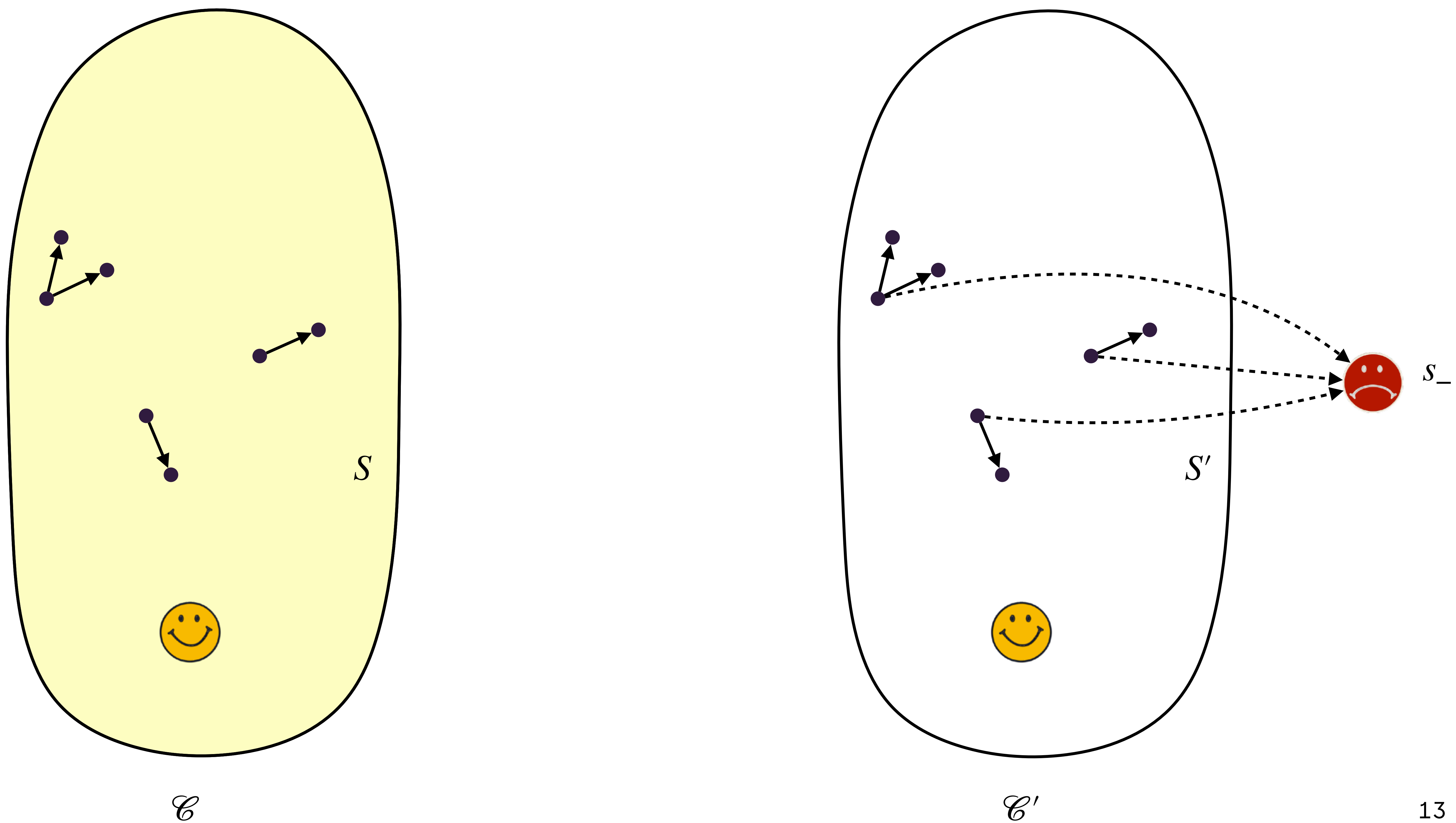
Importance sampling via an abstraction



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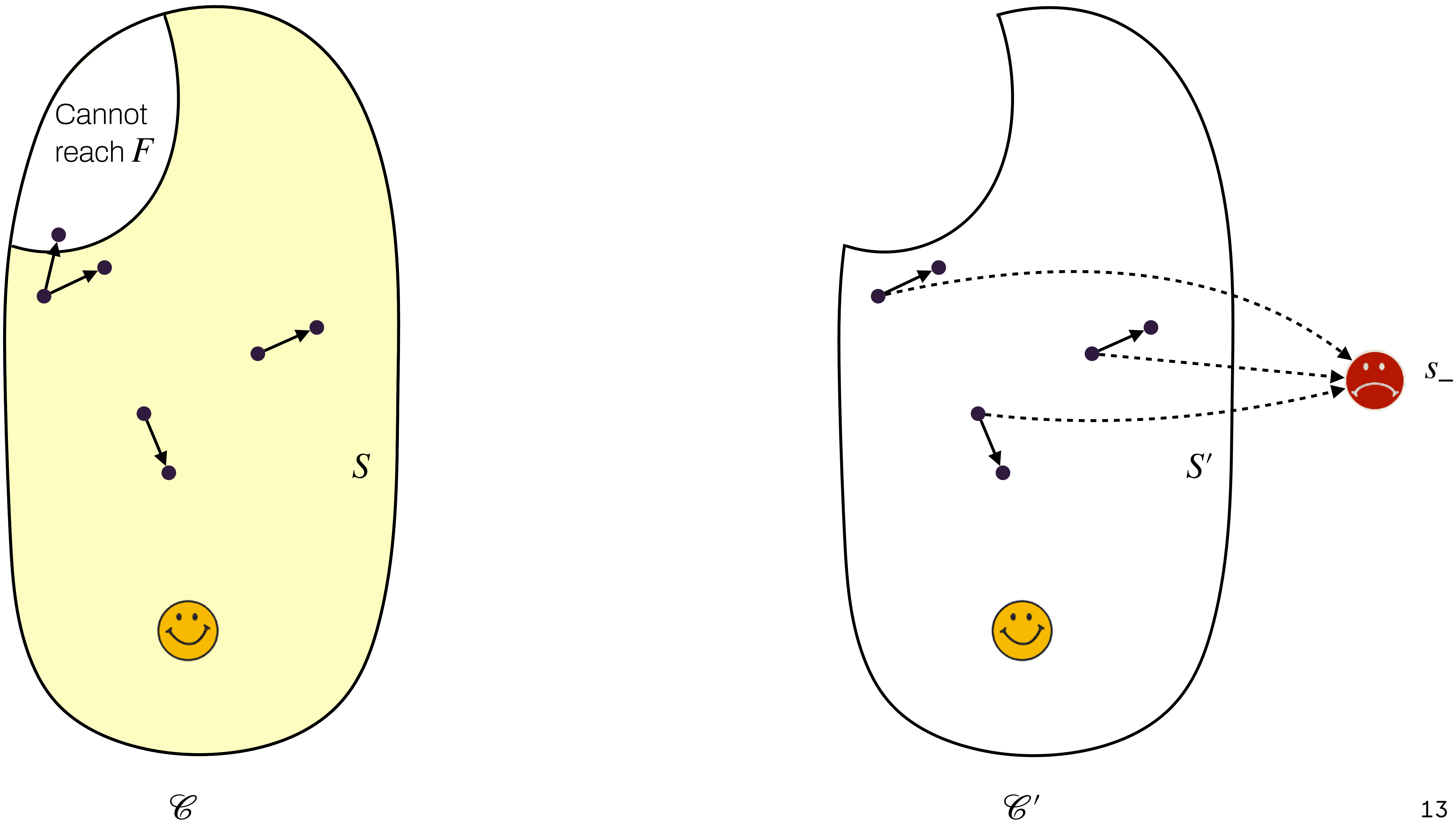

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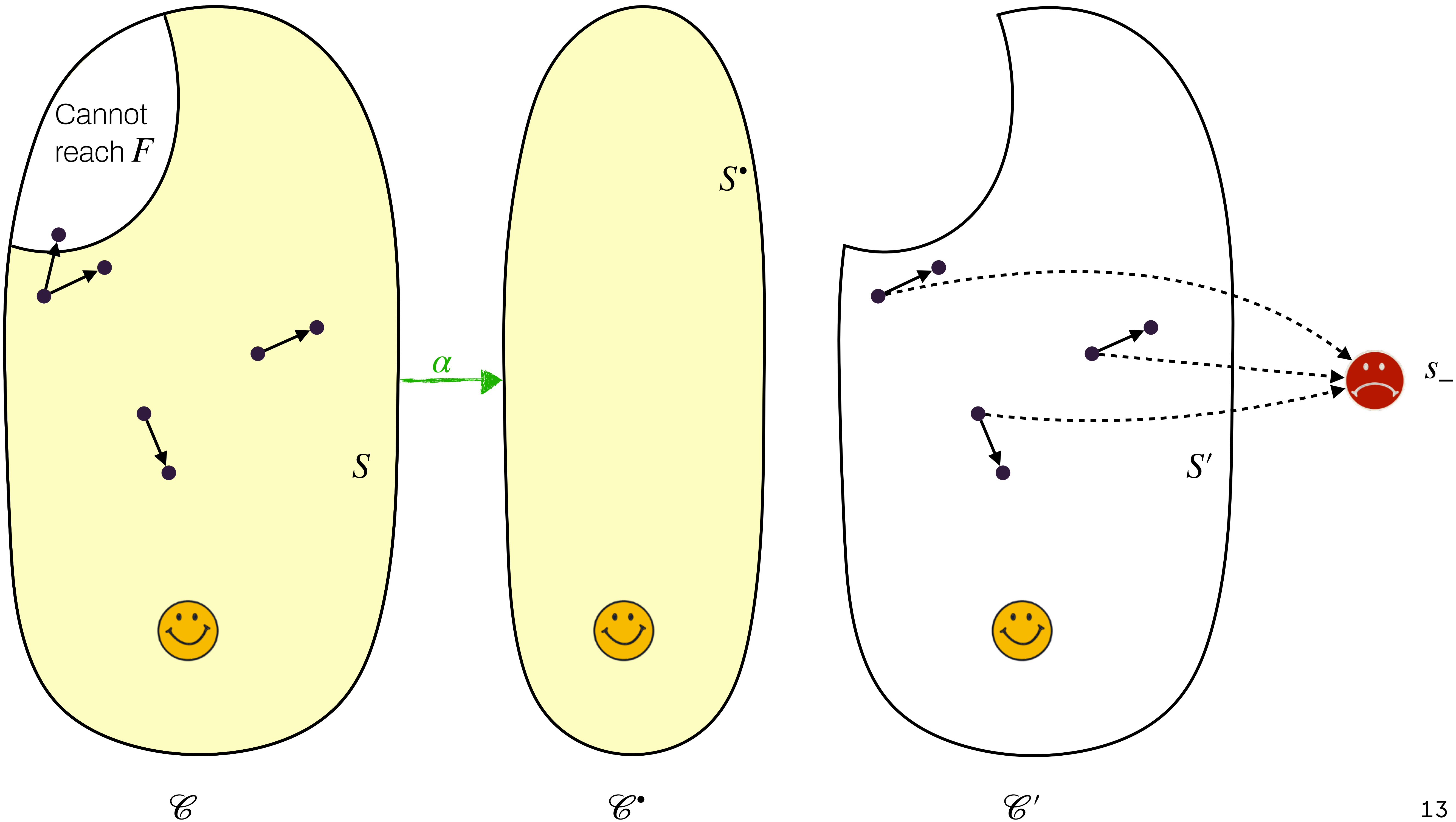

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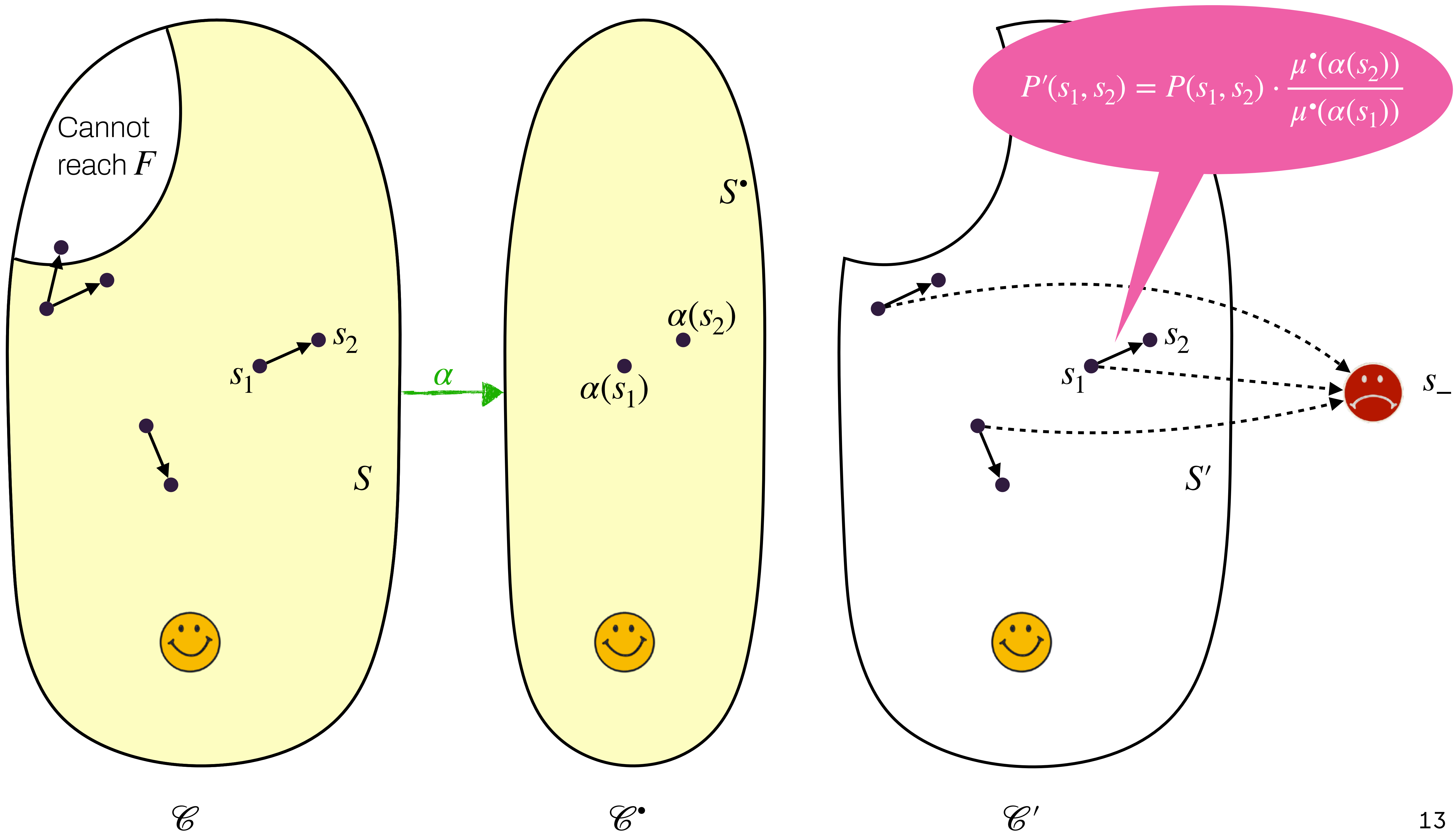

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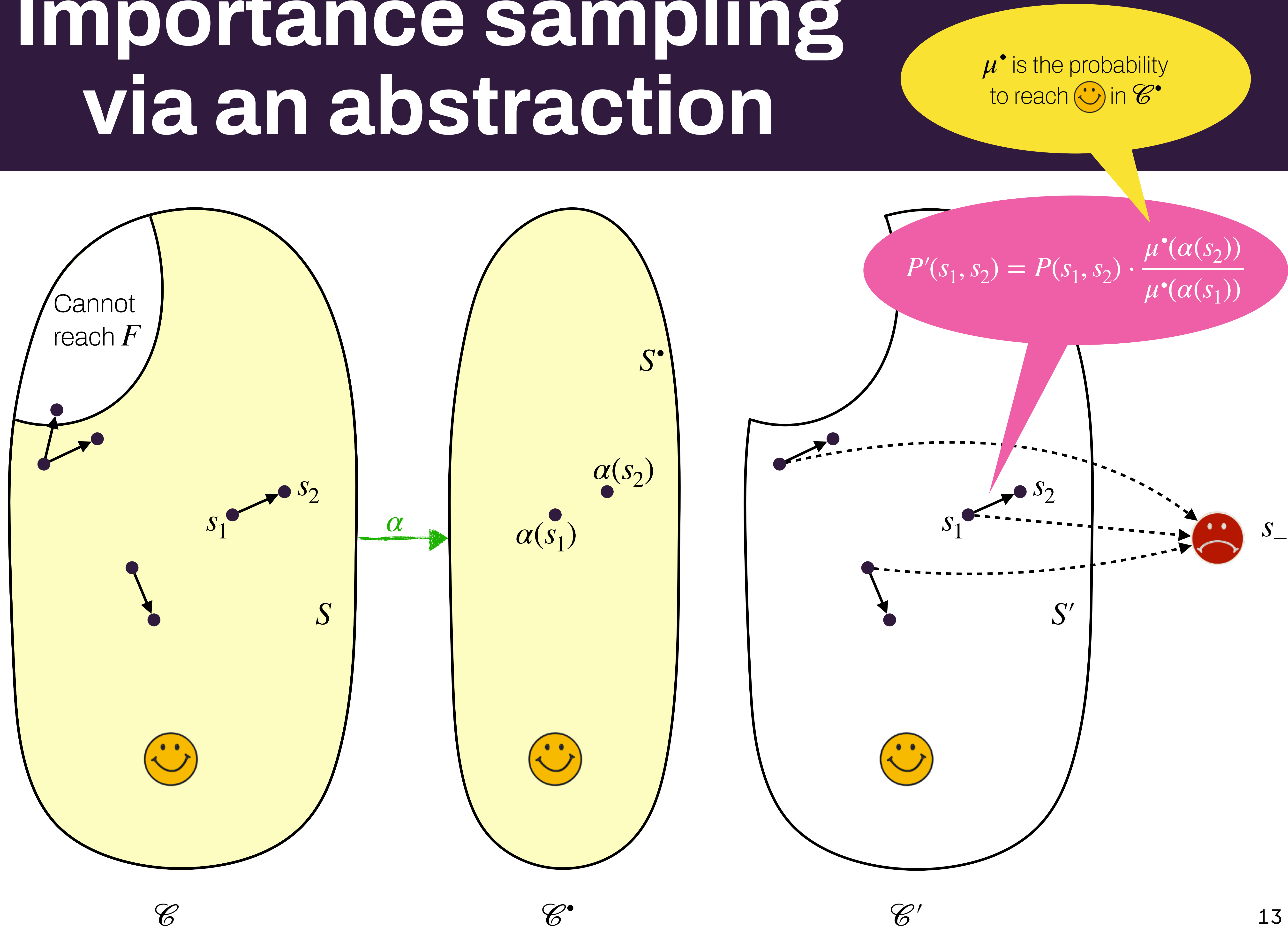

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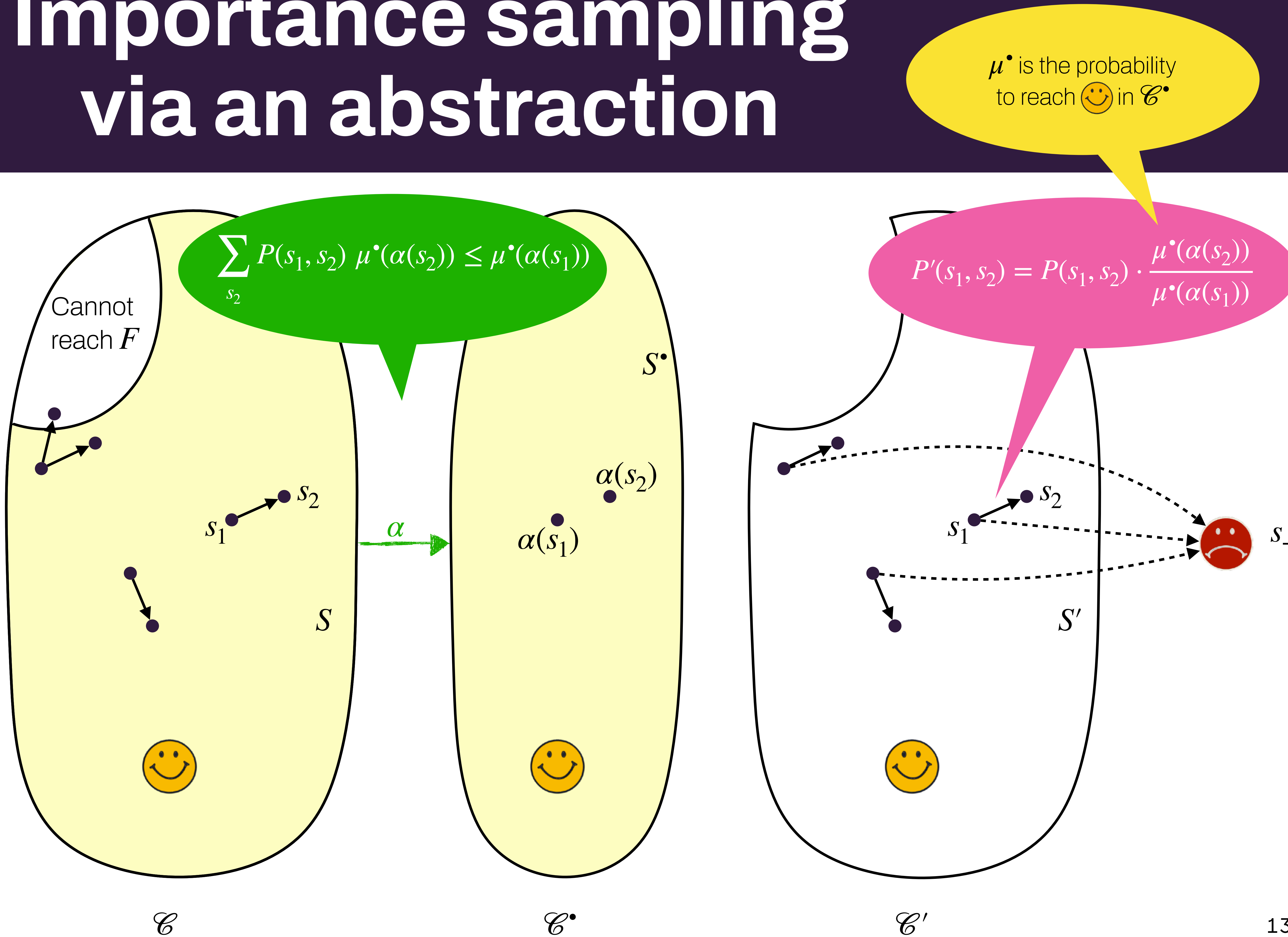

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- Model = layered Markov chain (LMC) $\mathscr{C}$: there is a level function $\lambda : S \to \mathbb{N}$ s.t.
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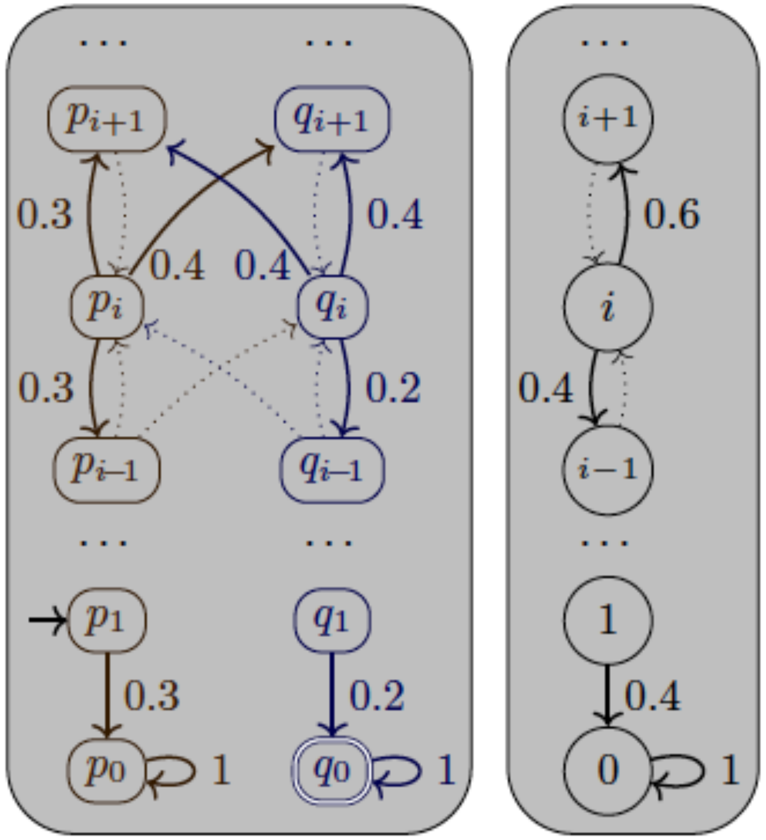

Correctness of the approach

Theorem

Let $\mathscr{C}$ be an LMC with level function λ, $\mathscr{C}_p^\bullet$ the random walk of parameter p. Assume there is N_0 s.t. $\frac{1}{2} < p < p_+ = \inf\{P^+(s) \mid \lambda(s) > N_0\}$. Then:

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Congestion

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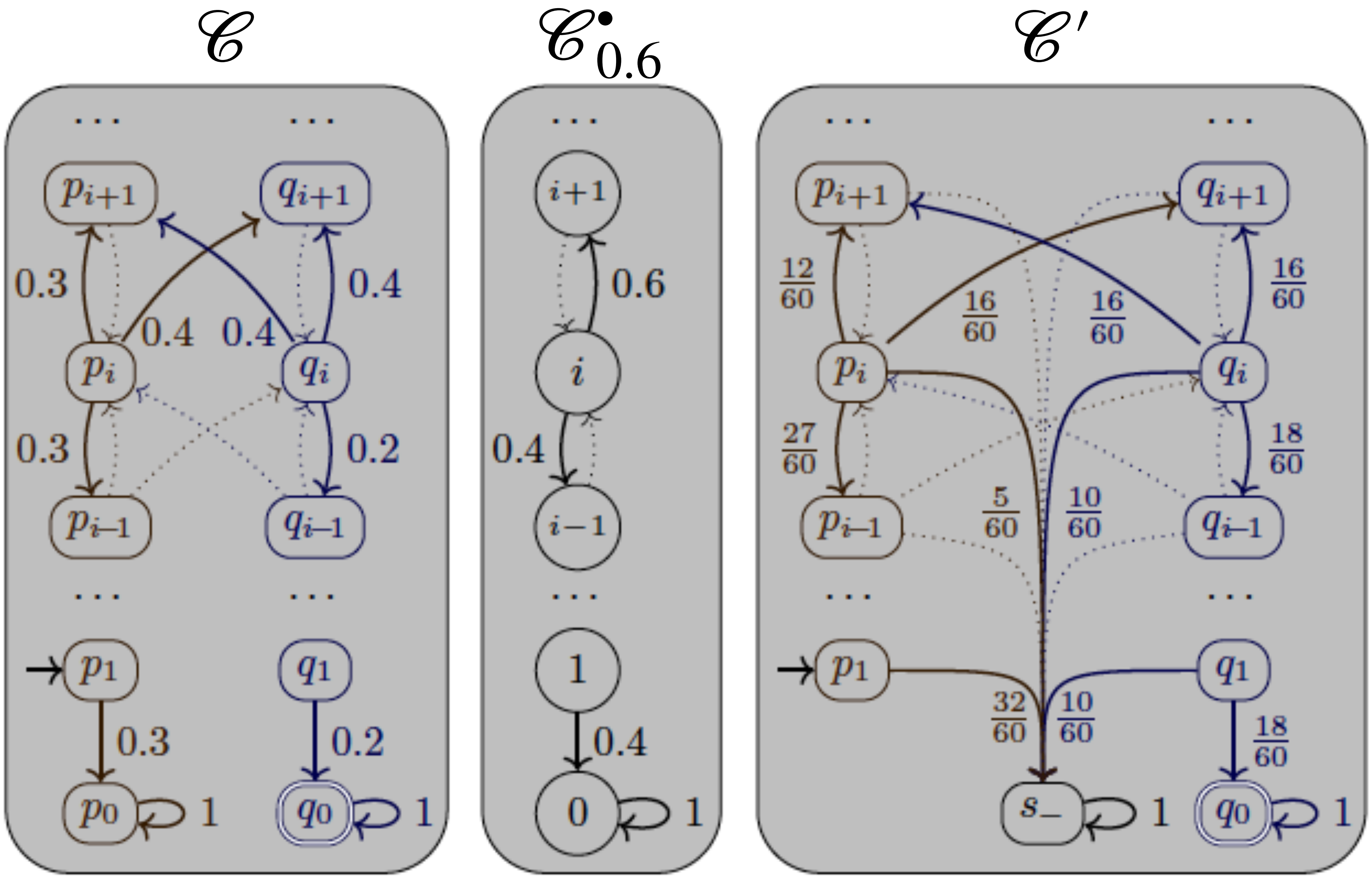
Example



$\mathscr{C}$ is not decisive

$\mathscr{C}'$ is decisive

Implementation and experiments

https://cosmos.lacl.fr/

[BBDHP15] P. Ballarini, B. Barbot, M. Duflot, S. Haddad, N. Pekergin. Hasl: A new approach for performance evaluation and model checking from concepts to experimentation (Performance Evaluation)

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- Implementation of the two approaches in tool **Cosmos** (development effort: Benoît Barbot)
- Application to probabilistic pushdown automata viewed as LMCs
- Methodology:
  - If $\mathscr{C}$ is decisive
    - Apply Approx and Estim on $\mathscr{C}$
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Note: in all experiments, the confidence is set to $99\ \%$

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First example

- State-free proba. pushdown automaton $\mathscr{C}$:$\{A \xrightarrow{1} C; A \xrightarrow{n} BB; B \xrightarrow{5} \varepsilon; B \xrightarrow{n} AA; C \xrightarrow{1} C\}$
- Start from A, and target the empty stack

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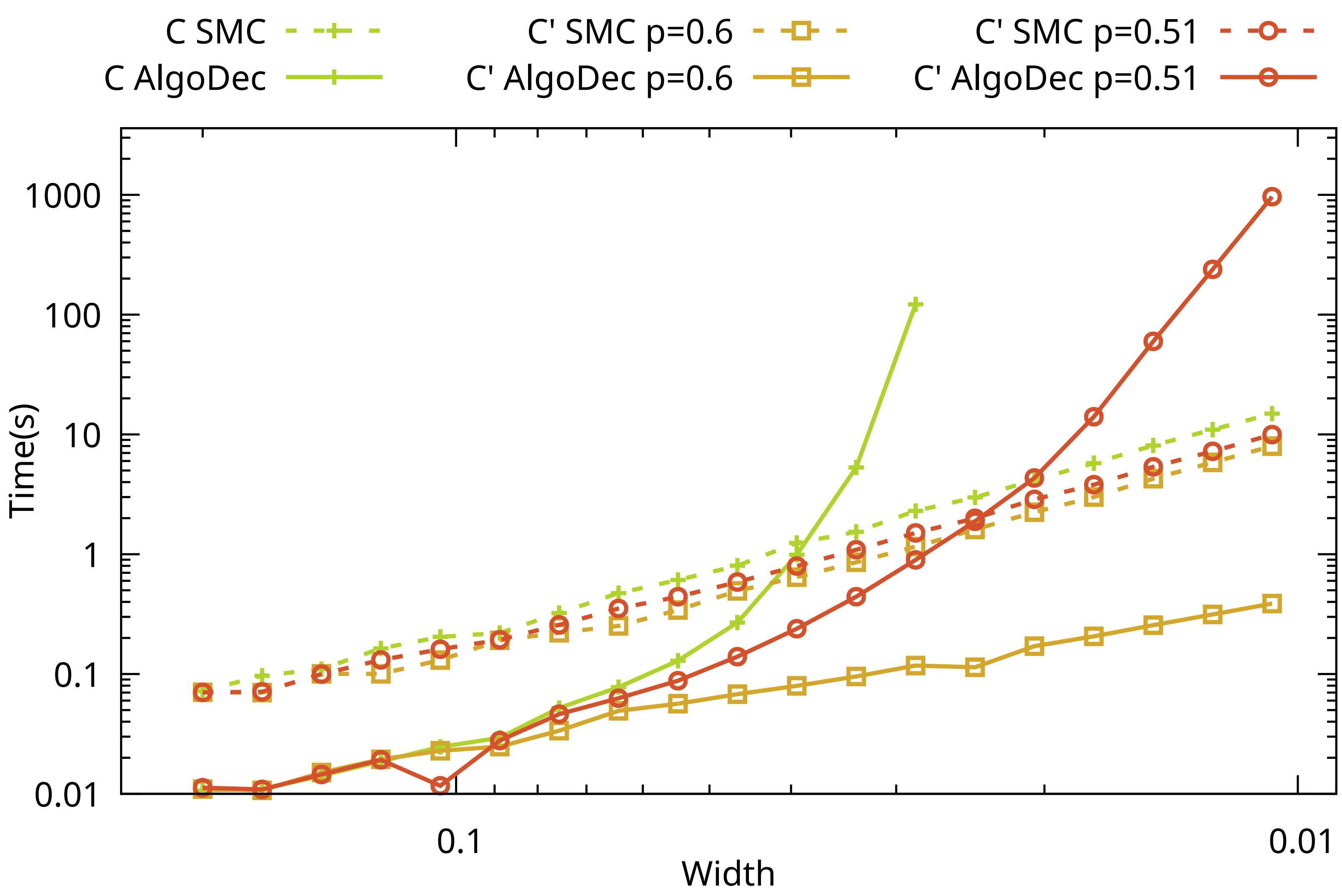

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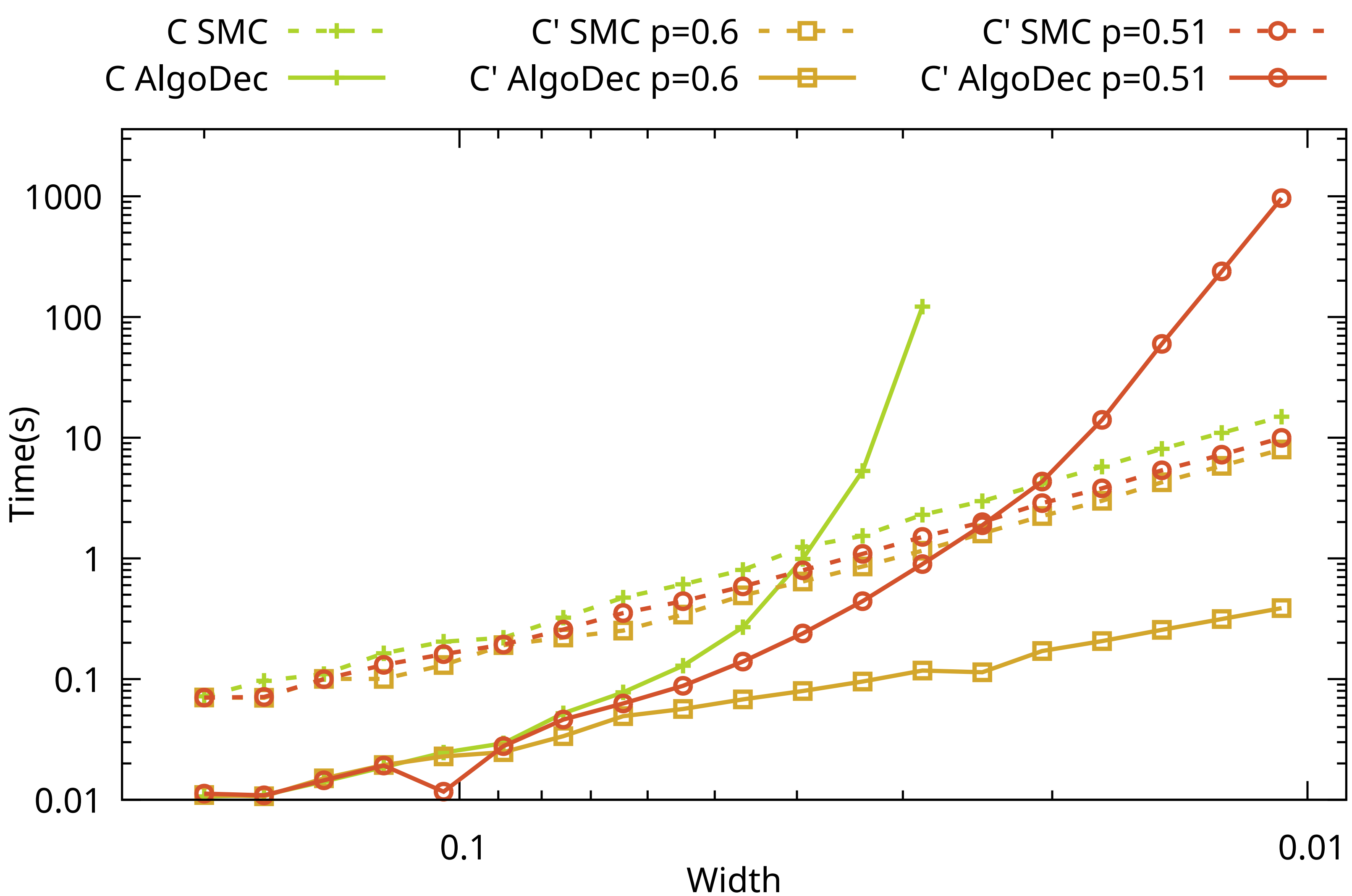
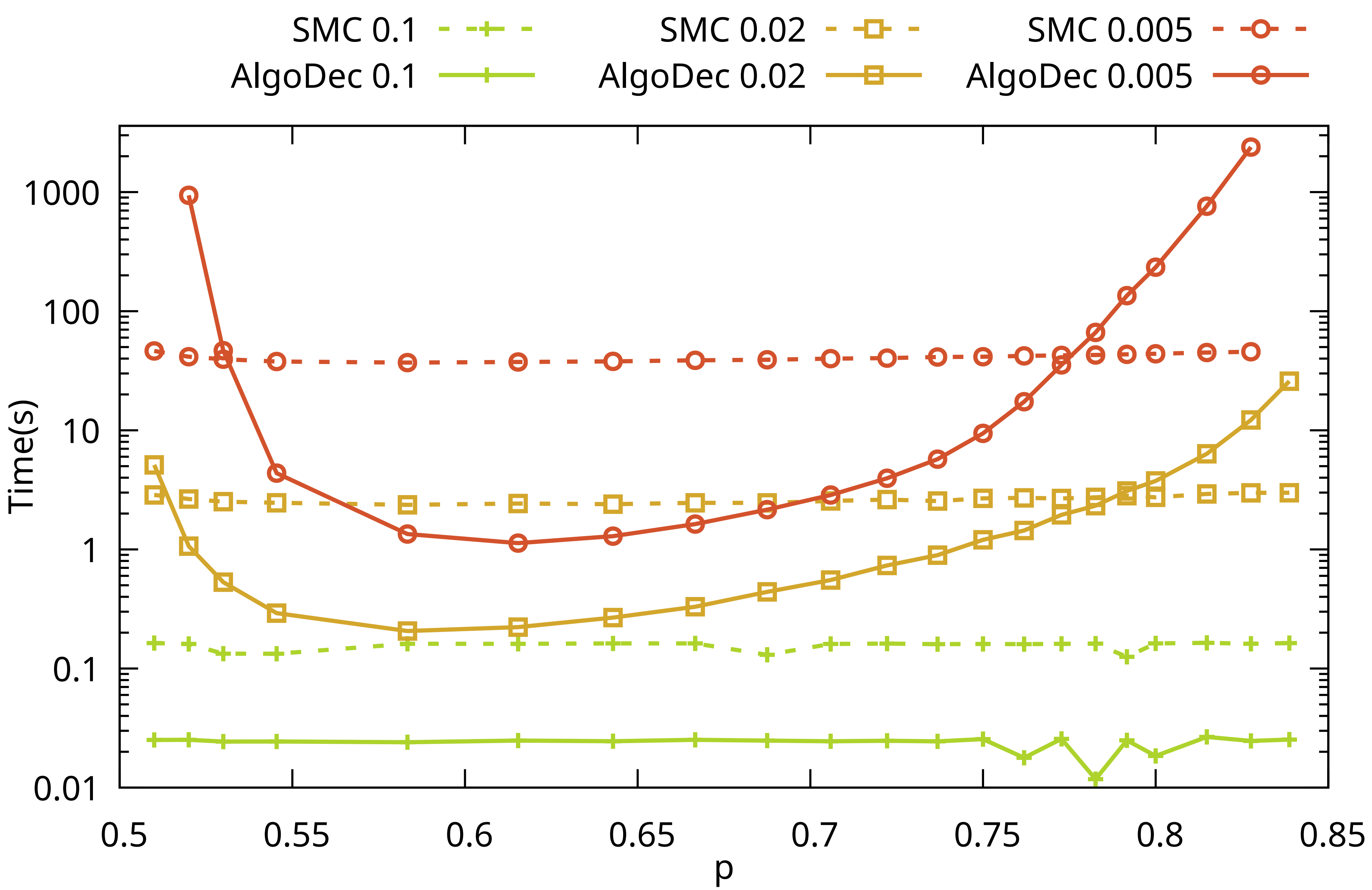


Precision ε

- In Estim (SMC): doubling the precision impacts in square on computation time (slope 2 in this log-log scale)
- Importance sampling seems to improve the analysis time, both for Approx and Estim (no formal guarantee, though)
- There seems to be « a best p » ($p = 0.6$ here)
- For that best p, Approx behaves very well!

First example — continued



Second example
Experimental results

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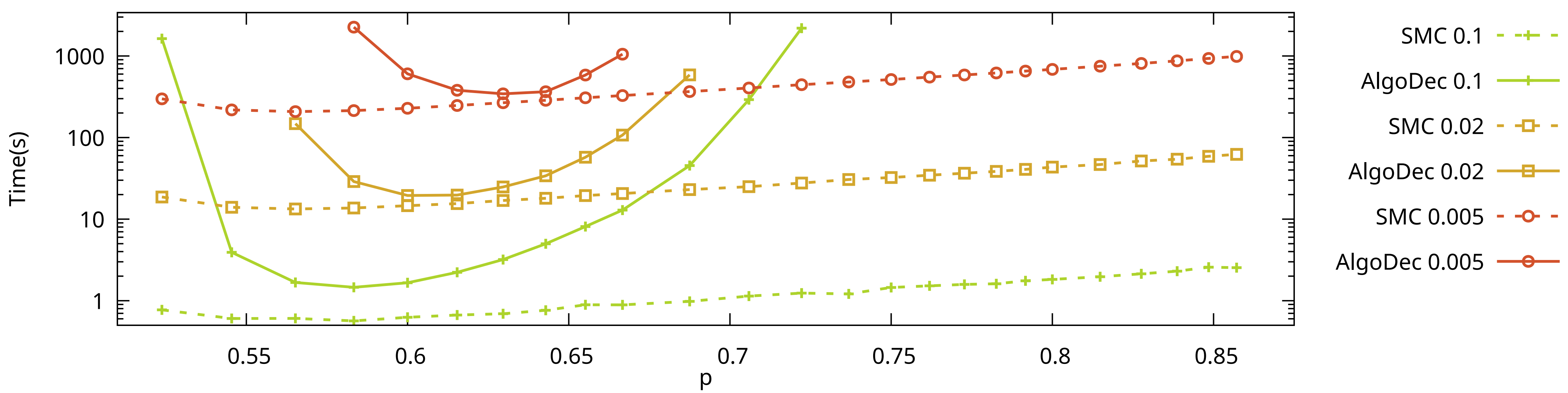

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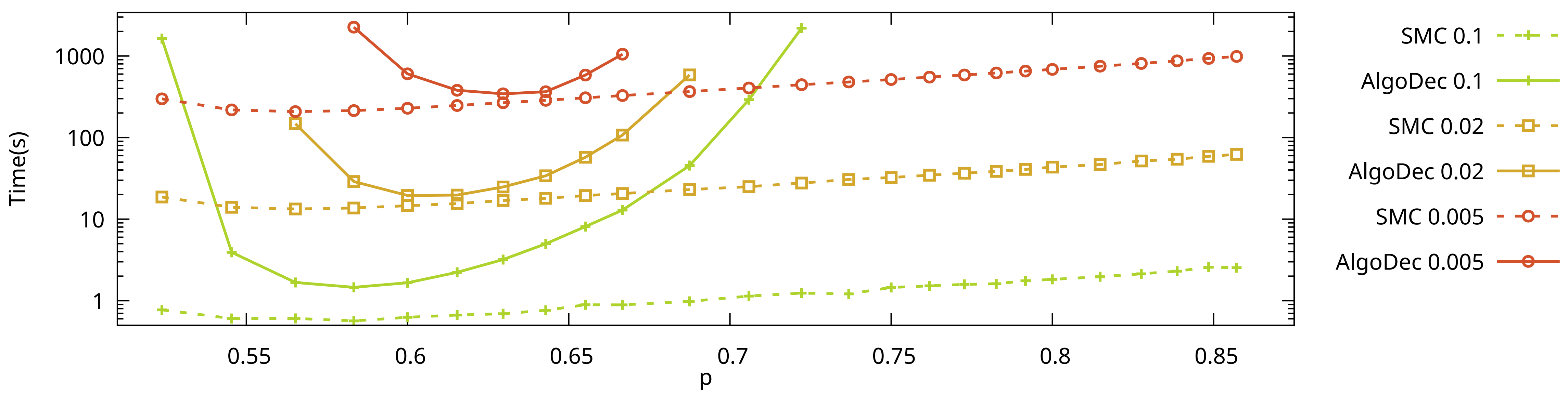

- Estim-SMC not too sensitive to p
  - Neverthess (log scale): clear bell effect on p
- Approx very sensitive to p

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Some more classes to be applied?

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