Laboratoire Méthodes Formelles

Beyond Decisiveness: When Statistical Verification Meets Numerical Verification

Benoît Barbot (LACL), Patricia Bouyer (LMF), Serge Haddad (LMF)

Supported by ANR projects MAVeriQ and BisoUS
(submitted, not yet on ArXiV)

Purpose of this work

Design algorithms to estimate probabilities in some **infinite-state** Markov chains, **with guarantees**

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Our contributions

- Review two existing approaches (approximation algorithm and estimation algorithm) and specify the required hypothesis for correctness
- Propose an approach based on **importance sampling** and **abstraction** to partly relax the hypothesis
- Analyze empirically the approaches

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Discrete-time Markov chain (DTMC)

$\mathscr{C} = (S, s_0, \delta)$ with S at most denumerable, $s_0 \in S$ and $\delta : S \to \mathrm{Dist}(S)$

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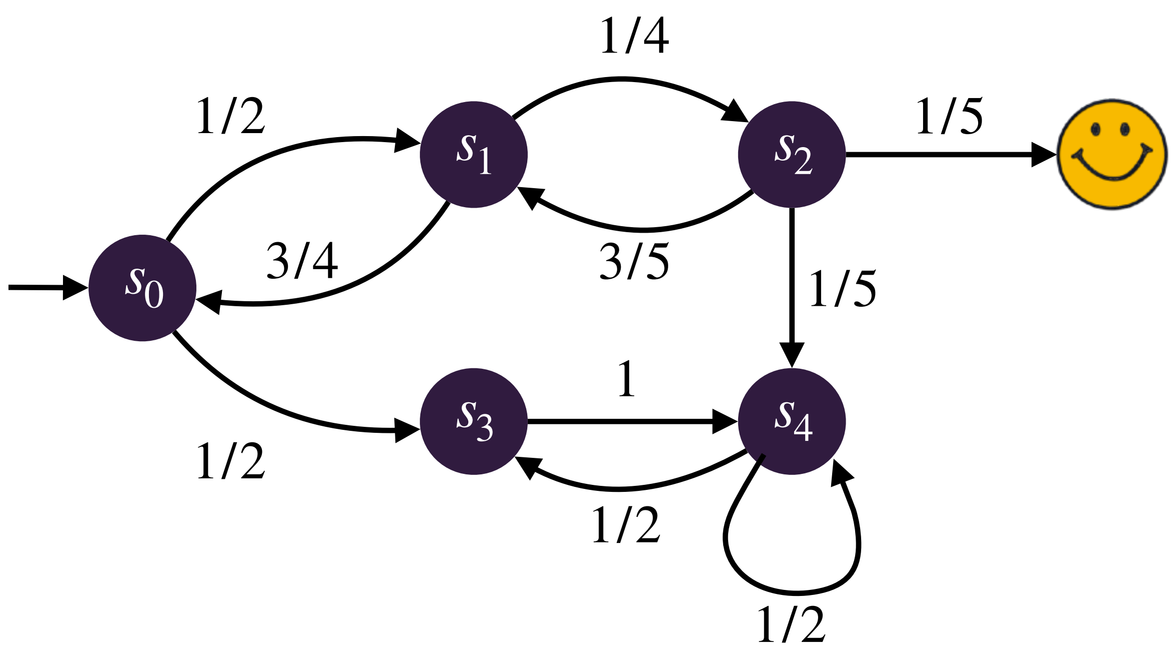

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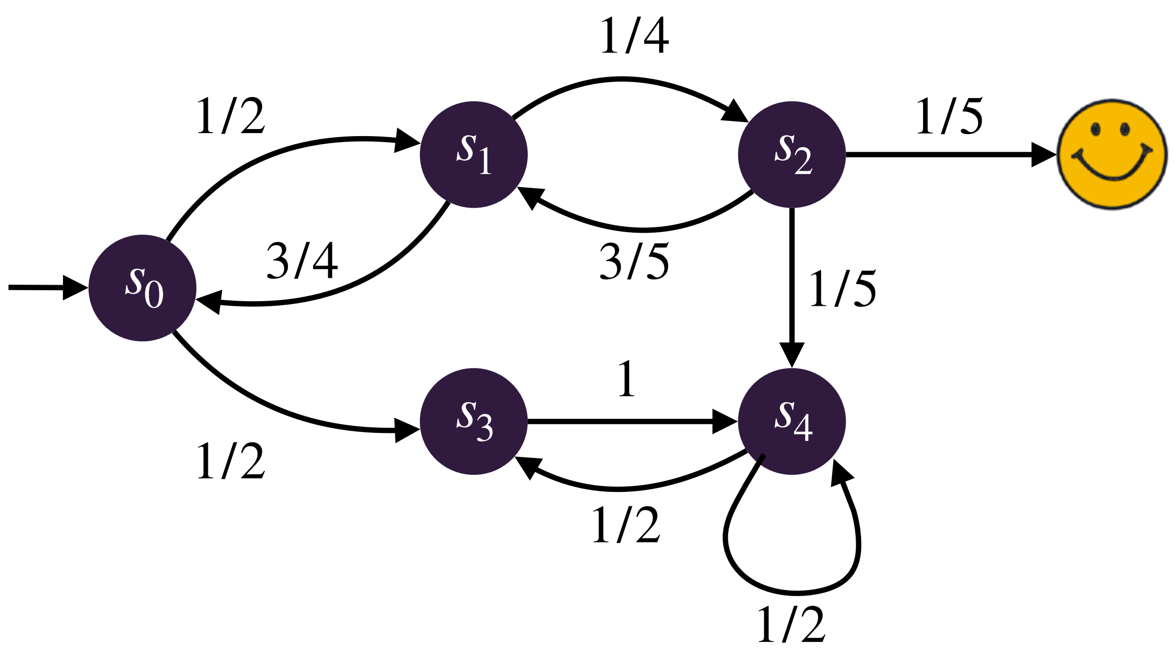

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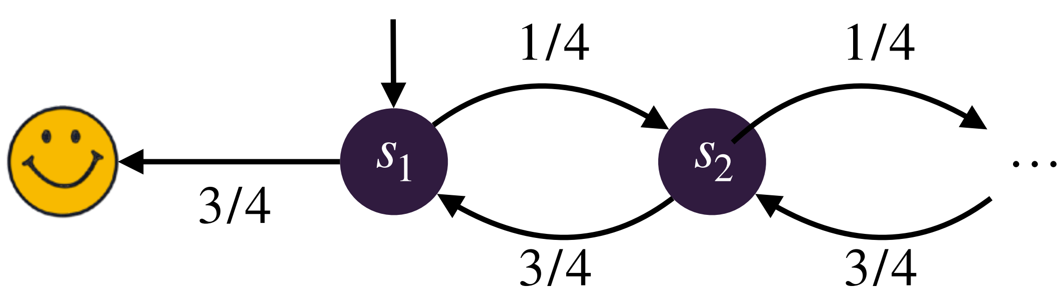

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(random walk of parameter $1/4$)

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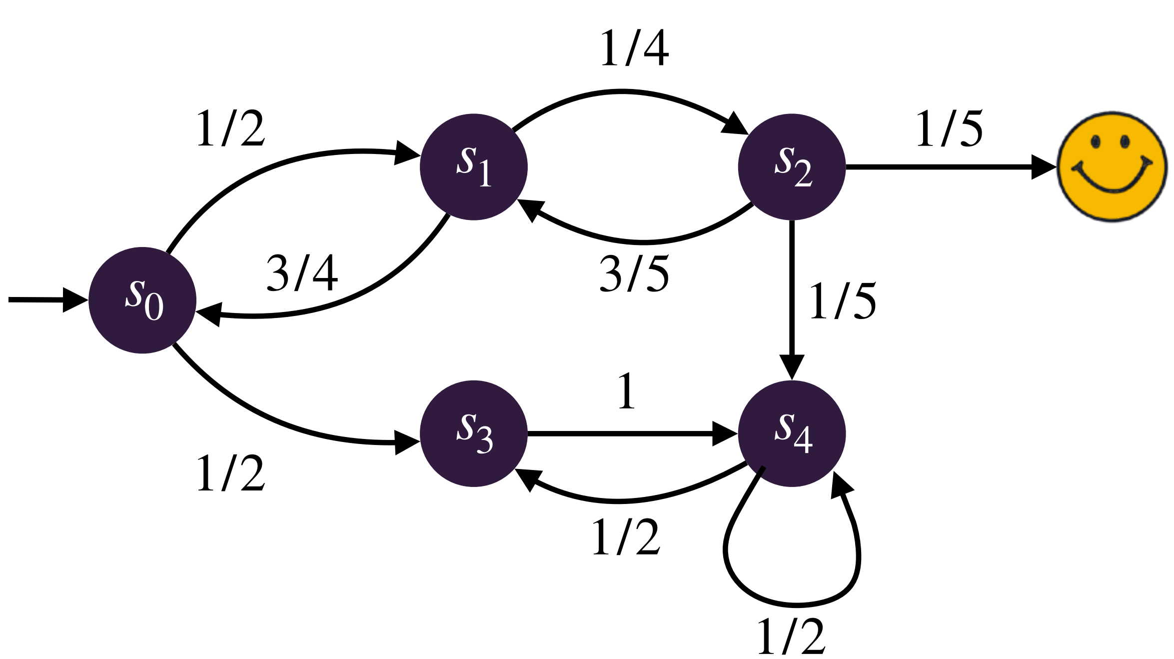

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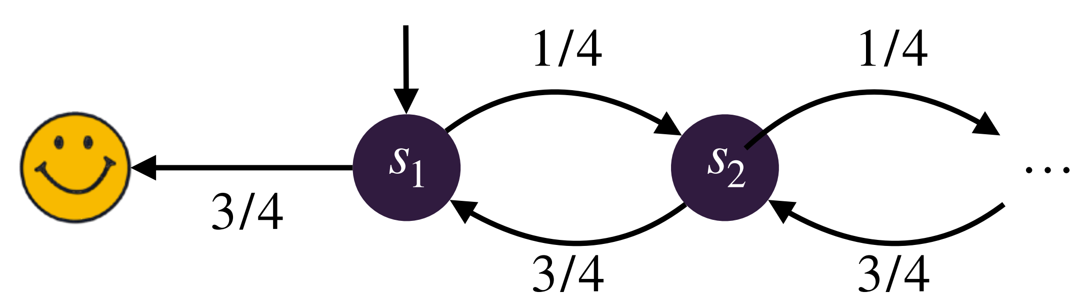

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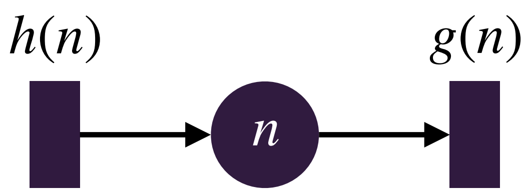
High-level models for (infinite) Markov chains

- Queues



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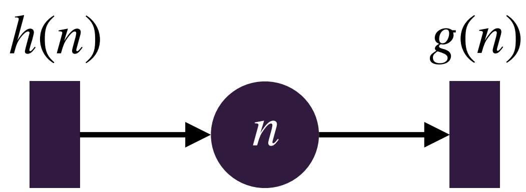

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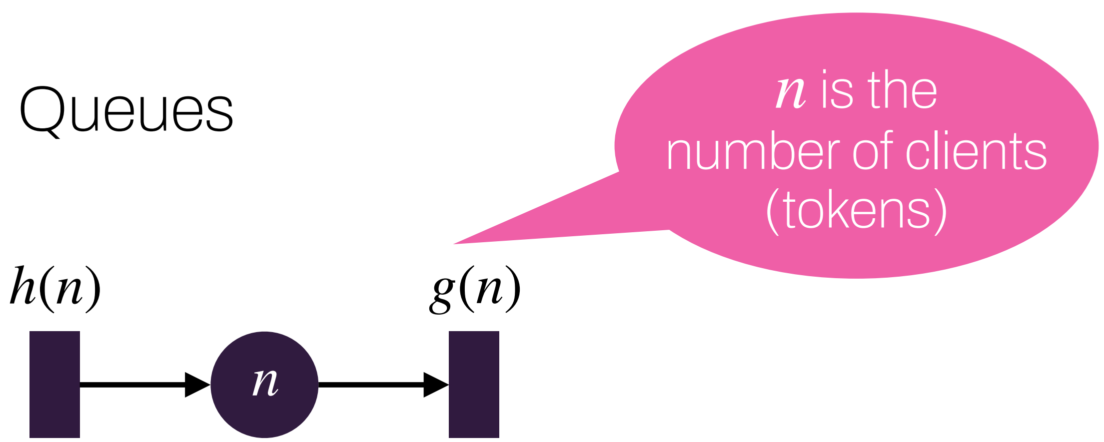

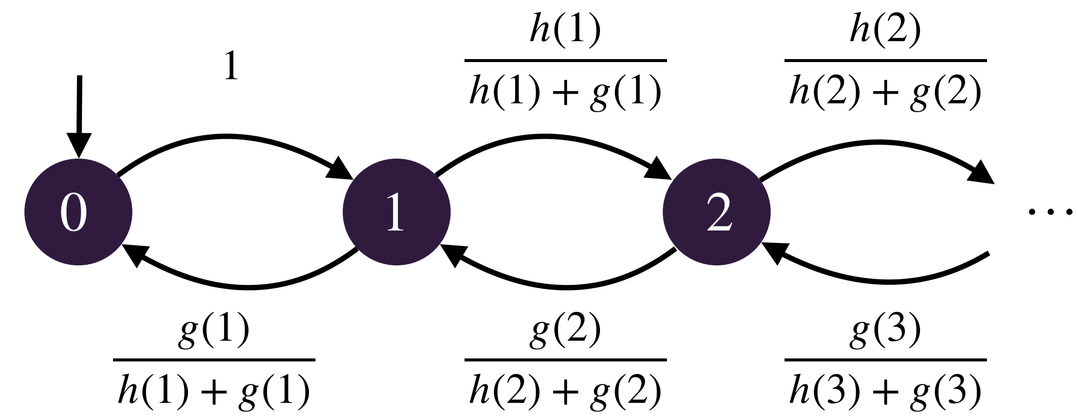

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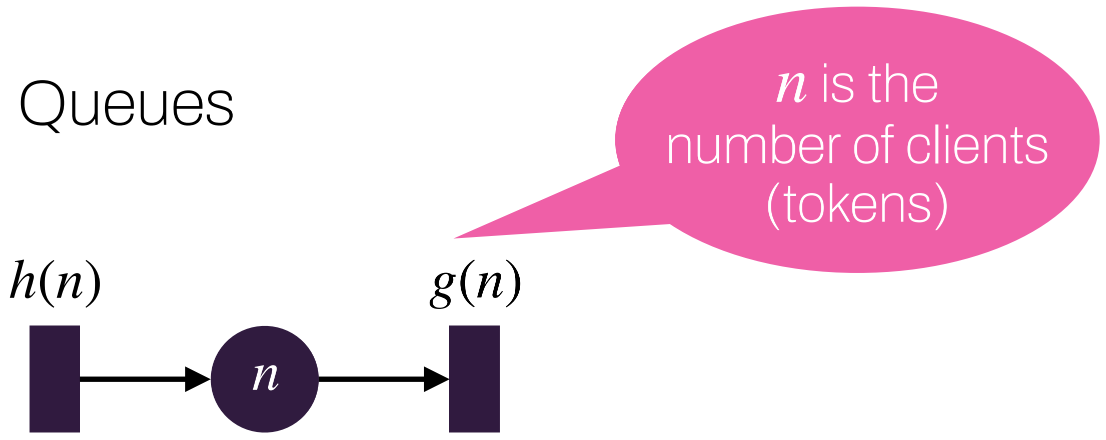

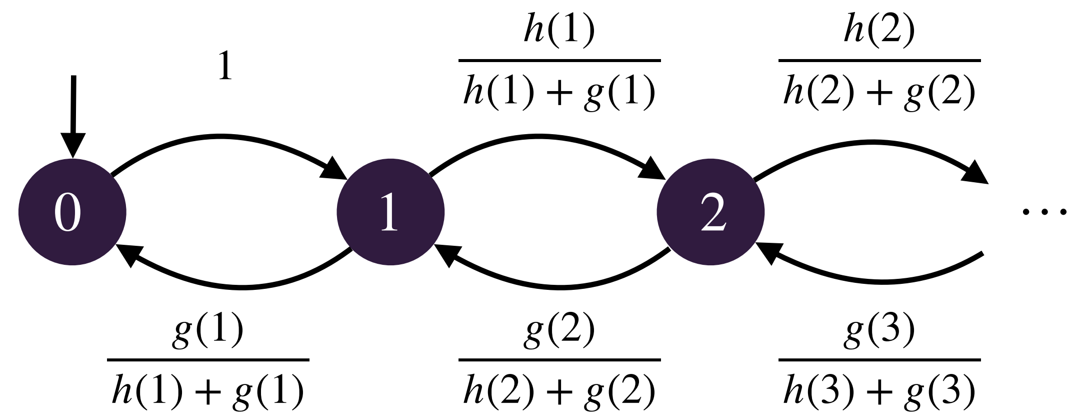

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$$A \xrightarrow{1} C \quad A \xrightarrow{n} BB \quad B \xrightarrow{5} \varepsilon$$
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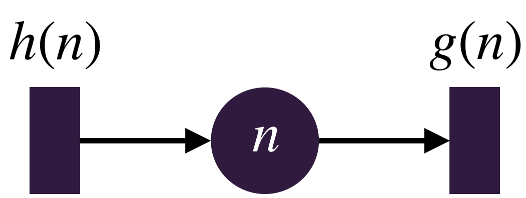

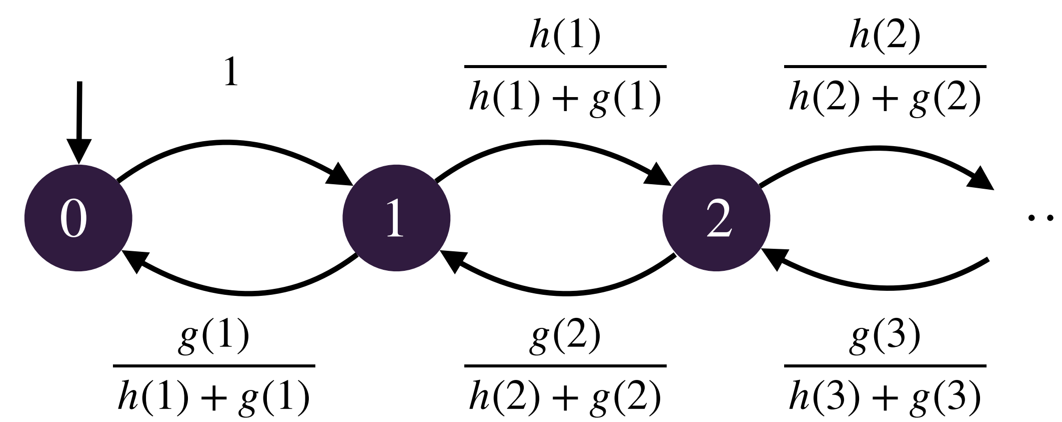

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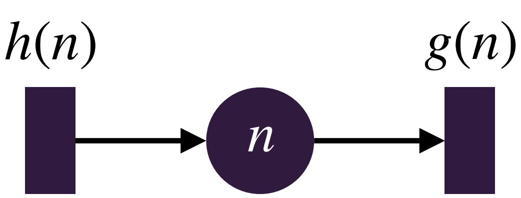

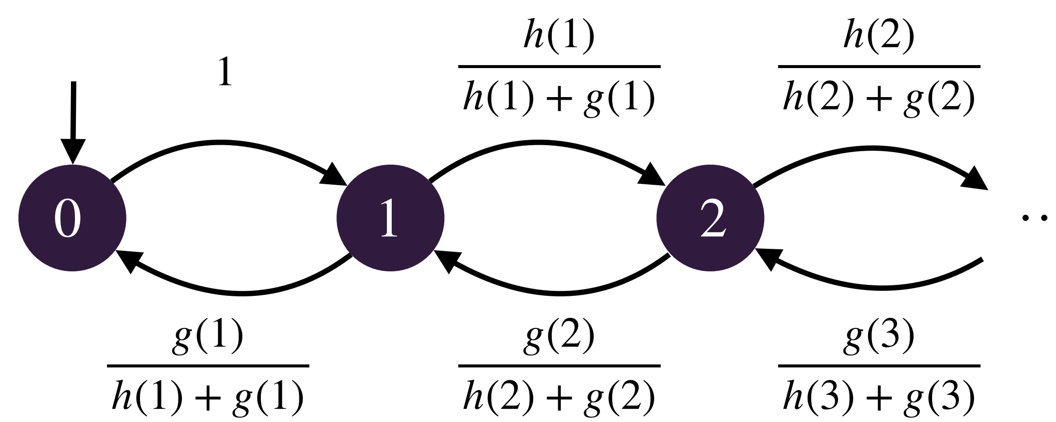

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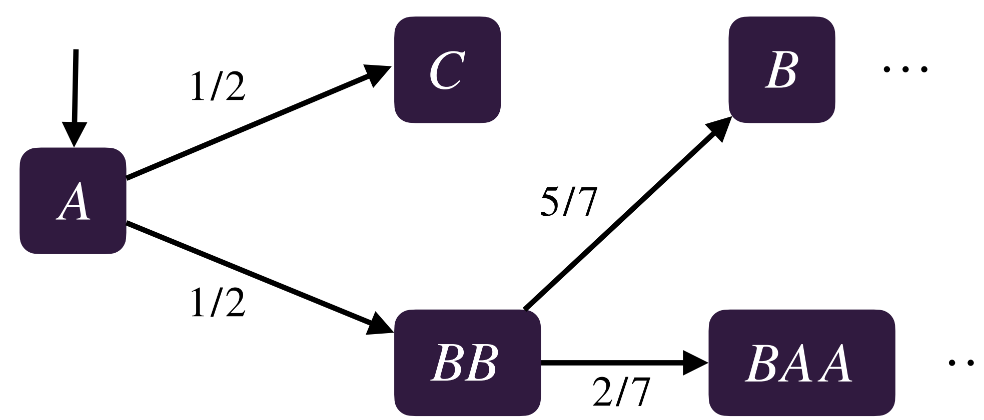

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Quantitative analysis of Markov chains

- Aim: compute the probability of property $\mathbf{F}$ ☺

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Very useful even beyond reachability properties (decomposition in BSCCs)

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[RKNP04] J. Rutten, M. Kwiatkowska, G. Norman, D. Parker. *Mathematical techniques for analyzing concurrent and probabilistic systems (Monograph)*

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- Specific approaches for **decisive** Markov chains

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Decisiveness

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Decisiveness

A DTMC $\mathscr{C}$ is decisive from s w.r.t. 🙂 if $\mathbb{P}_s(\mathbf{F}$ 🙂 $\vee\, \mathbf{F}$ ☹️ $) = 1$

[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)

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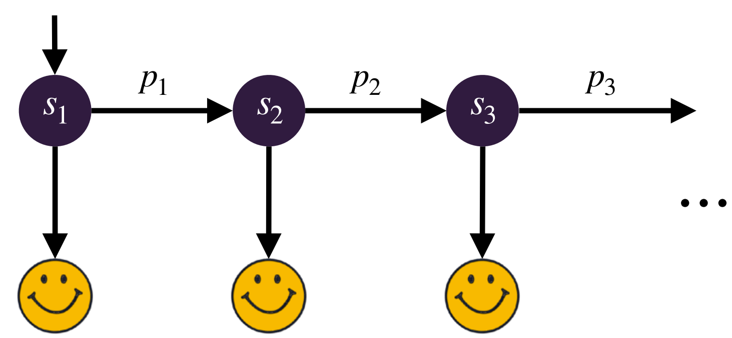

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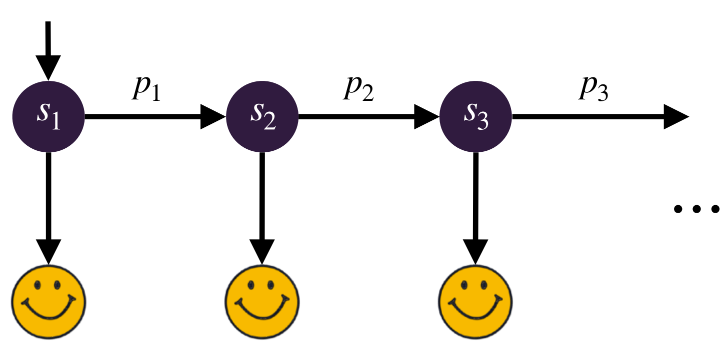

- $\mathbb{P}(\mathbf{G}\neg$ 🙂 $) = \prod_{i \geq 1} p_i$
- Decisive iff this product equals 0

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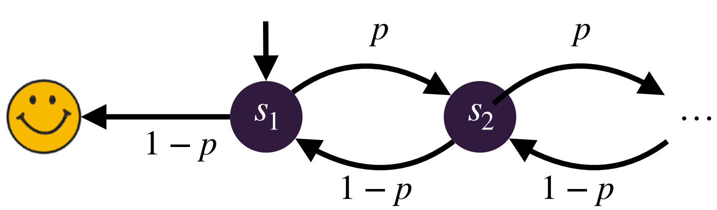

- Recurrent random walk ($p \leq 1/2$): decisive
- Transient random walk ($p > 1/2$): not decisive

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Deciding decisiveness?

Classes where decisiveness can be decided

- Probabilistic pushdown automata with constant weights [ABM07]
- Random walks with polynomial weights [FHY23]
- So-called probabilistic homogeneous one-counter machines with polynomial weights (this extends the model of quasi-birth death processes) [FHY23]

[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)
[FHY23] A. Finkel, S. Haddad, L. Yé. About decisiveness of dynamic probabilistic models (CONCUR'23)

Approximation scheme

- Aim: compute probability of $\mathbf{F}$

[IN97] P. Iyer, M. Narasimha. Probabilistic lossy channel systems (TAPSOFT'97)

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until $p_n^{\text{yes}} + p_n^{\text{no}} \geq 1 - \varepsilon$

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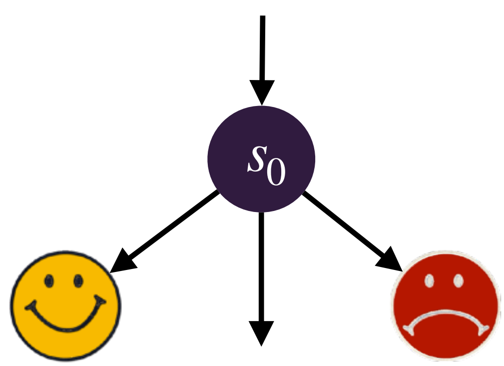

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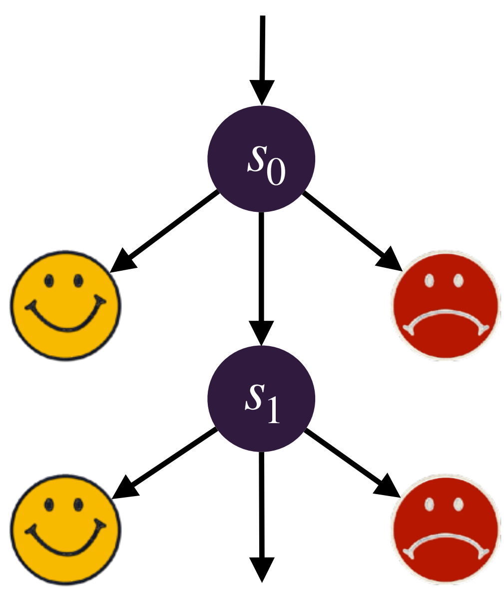

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$$\leq \qquad\qquad\qquad \geq$$

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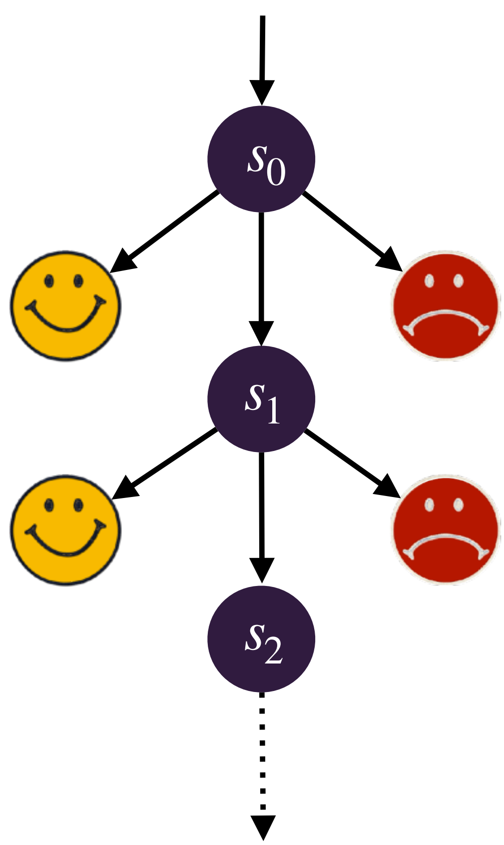

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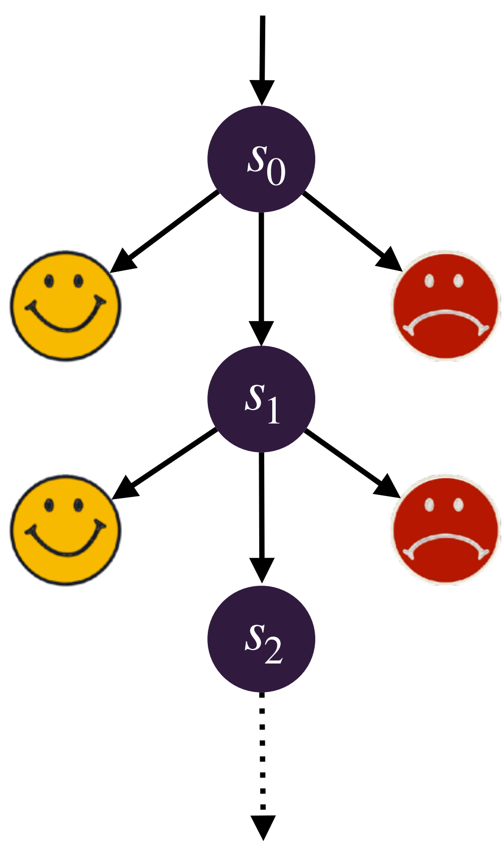

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$$\begin{array}{ccc} \text{|}\wedge & & \vee\text{|} \end{array}$$

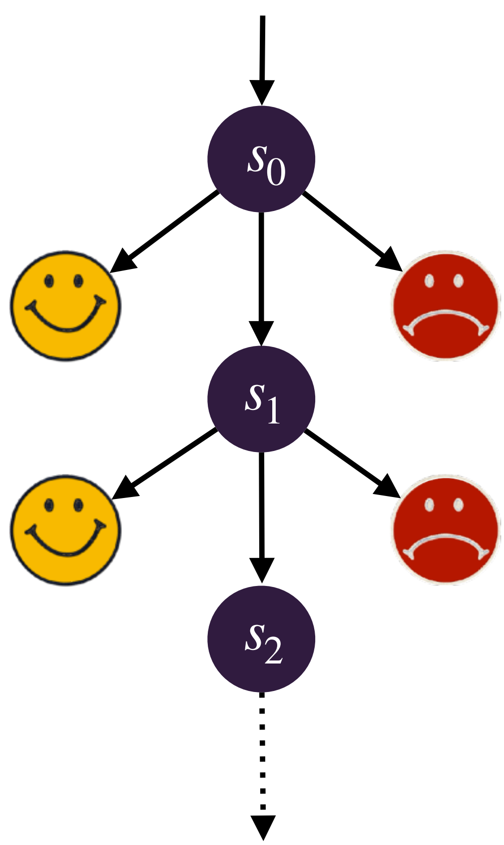
$$p_2^{\text{yes}} \leq \mathbb{P}(\mathbf{F} \text{☺}) \leq 1 - p_2^{\text{no}}$$

$$\begin{array}{ccc} \text{|}\wedge & \vdots & \vee\text{|} \end{array}$$

Does it converge?

Approximation scheme

- <u>Aim</u>: compute probability of $\mathbf{F}\,\smiley$
- $\frownie \;= \{s \in S \mid s \not\models \exists \mathbf{F}\,\smiley\}$



Approximation scheme

Given $\varepsilon > 0$, for every n, compute:

$$\begin{cases} p_n^{\text{yes}} & = \mathbb{P}(\mathbf{F}_{\leq n}\,\smiley) \\ p_n^{\text{no}} & = \mathbb{P}(\neg\,\smiley\,\mathbf{U}_{\leq n}\,\frownie) \end{cases}$$

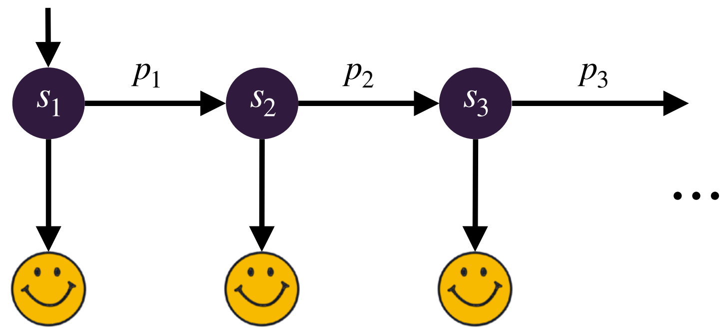
until $p_n^{\text{yes}} + p_n^{\text{no}} \geq 1 - \varepsilon$

$$\begin{array}{ccccc} p_1^{\text{yes}} & \leq & \mathbb{P}(\mathbf{F}\,\smiley) & \leq & 1 - p_1^{\text{no}} \\ \rotatebox{90}{$\leq$} & & & & \rotatebox{90}{$\geq$} \\ p_2^{\text{yes}} & \leq & \mathbb{P}(\mathbf{F}\,\smiley) & \leq & 1 - p_2^{\text{no}} \\ \rotatebox{90}{$\leq$} & & \vdots & & \rotatebox{90}{$\geq$} \end{array}$$

$$\lim_{n \to \infty} p_n^{\text{Yes}} = \mathbb{P}(\mathbf{F}\,\smiley)$$

[IN97] P. Iyer, M. Narasimha. Probabilistic lossy channel systems (TAPSOFT'97)

Non-converging example



with $\prod_{i \geq 1} p_i > 0$

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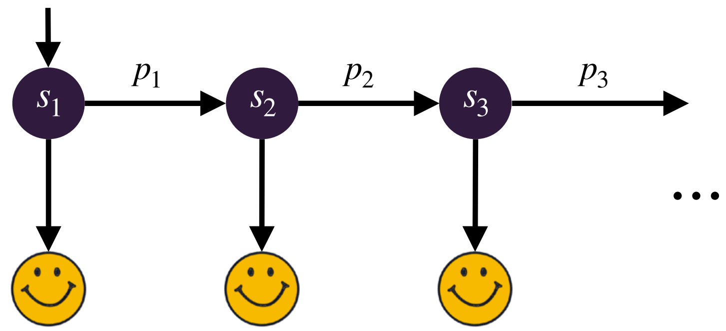

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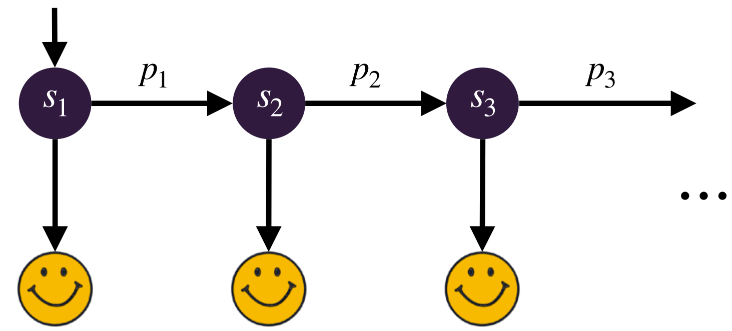
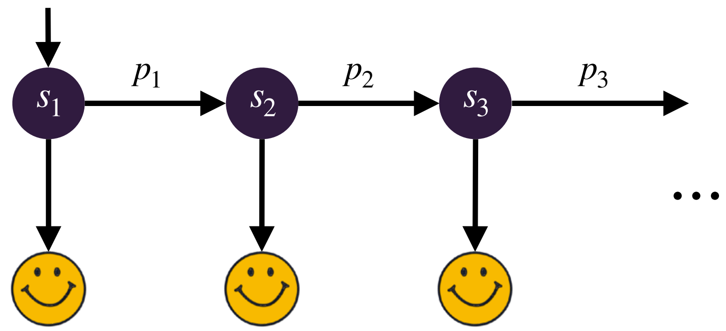


with $\prod_{i \geq 1} p_i > 0$

-  $= \emptyset$

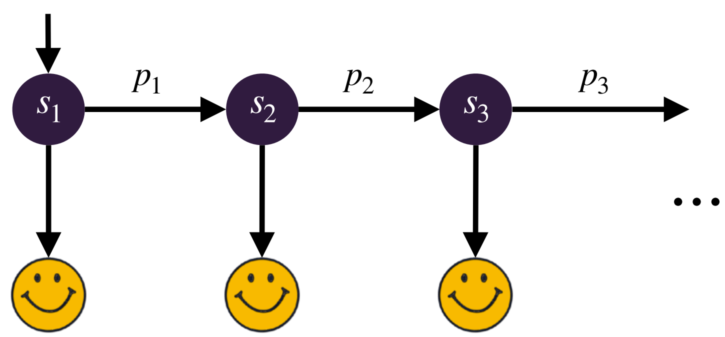
Non-converging example



with $\prod_{i\geq 1} p_i > 0$

-  $= \varnothing$
- $\lim_{n\to+\infty} p_n^{\text{yes}} = \mathbb{P}(\mathbf{F}$  $) < 1$

Non-converging example



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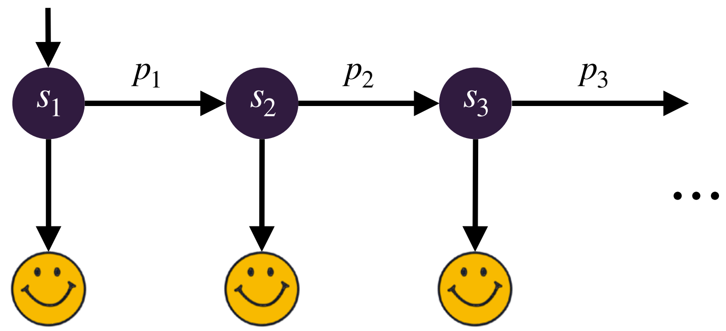

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The approximation scheme
does not converge

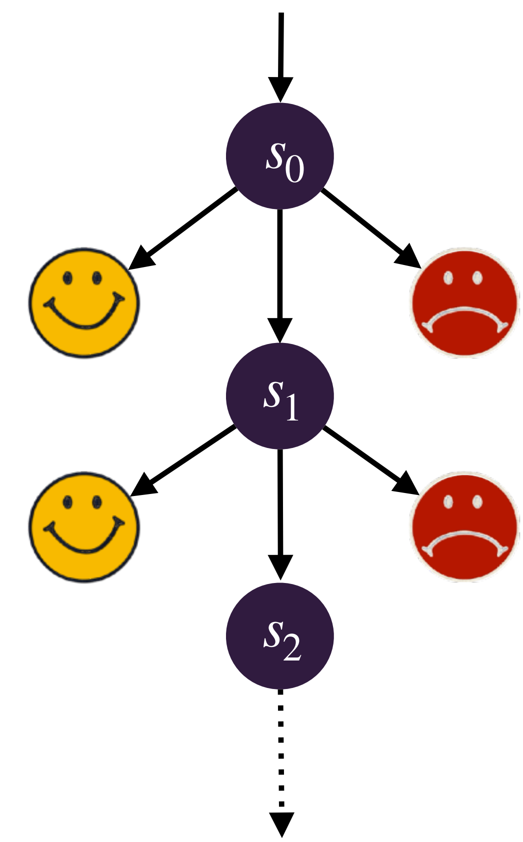
Termination of the approx. scheme

Approximation scheme

Given $\varepsilon > 0$:

$$\begin{cases} p_n^{\text{yes}} & = & \mathbb{P}(\mathbf{F}_{\leq n} \smiley) \\ p_n^{\text{no}} & = & \mathbb{P}(\neg \smiley \, \mathbf{U}_{\leq n} \frownie) \end{cases}$$

until $p_n^{\text{yes}} + p_n^{\text{no}} \geq 1 - \varepsilon$



[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)

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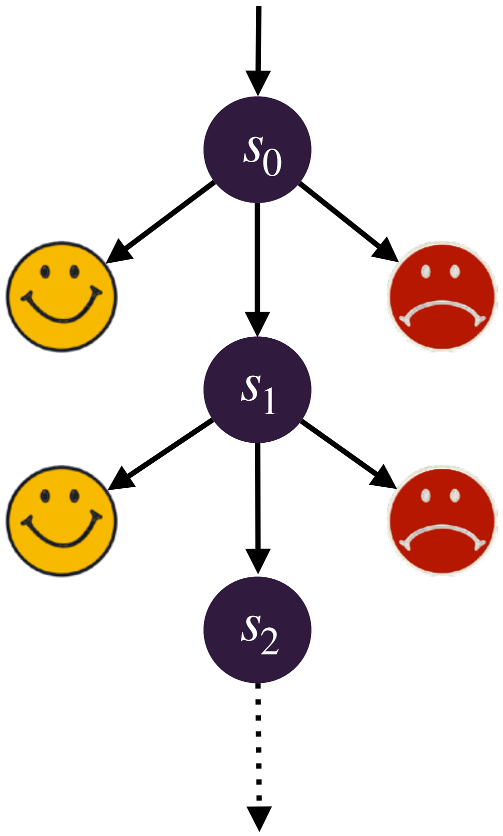

$\mathscr{C}$ is decisive from s_0 w.r.t. ☺
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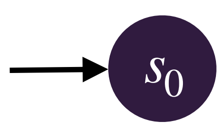
Statistical model-checking

Sample N paths



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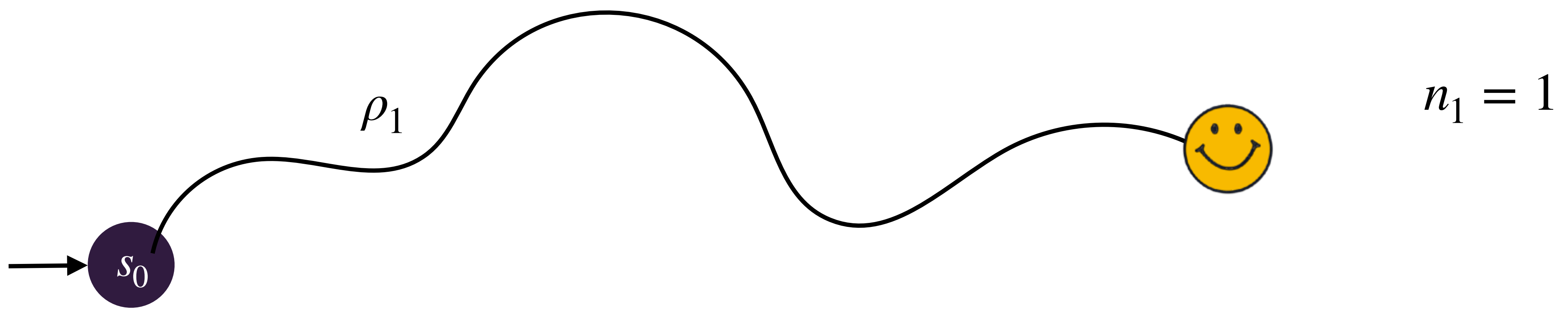

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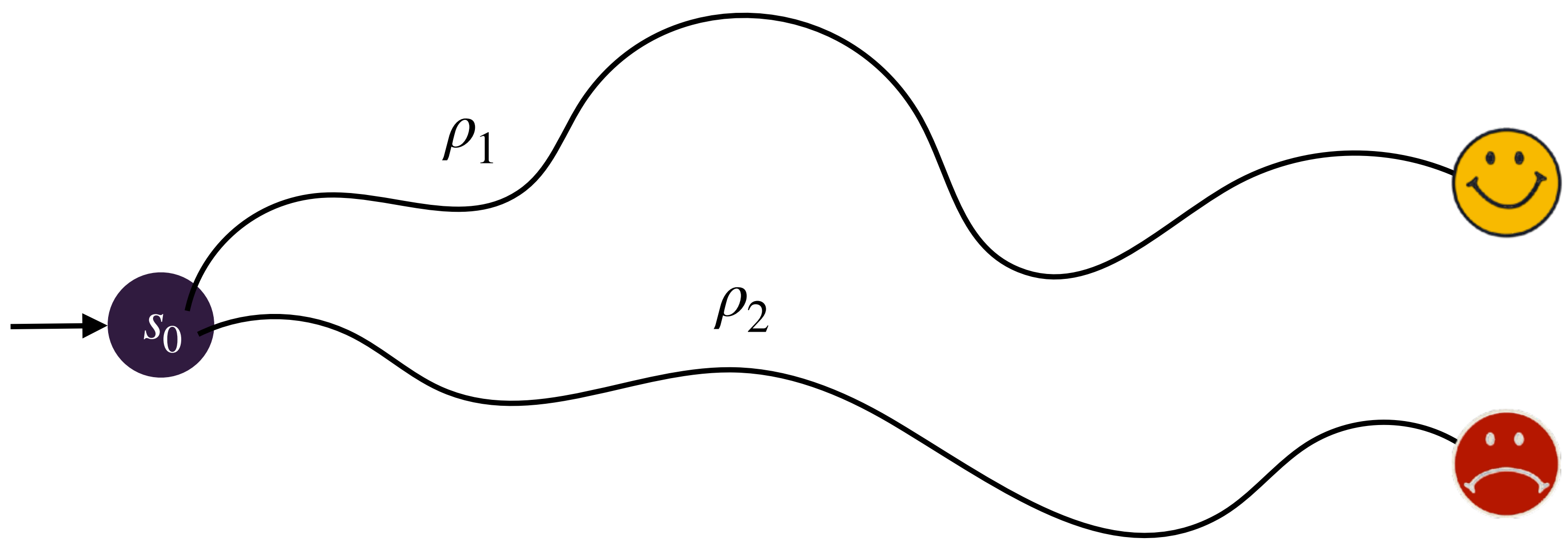

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$n_2 = n_1$

Statistical model-checking

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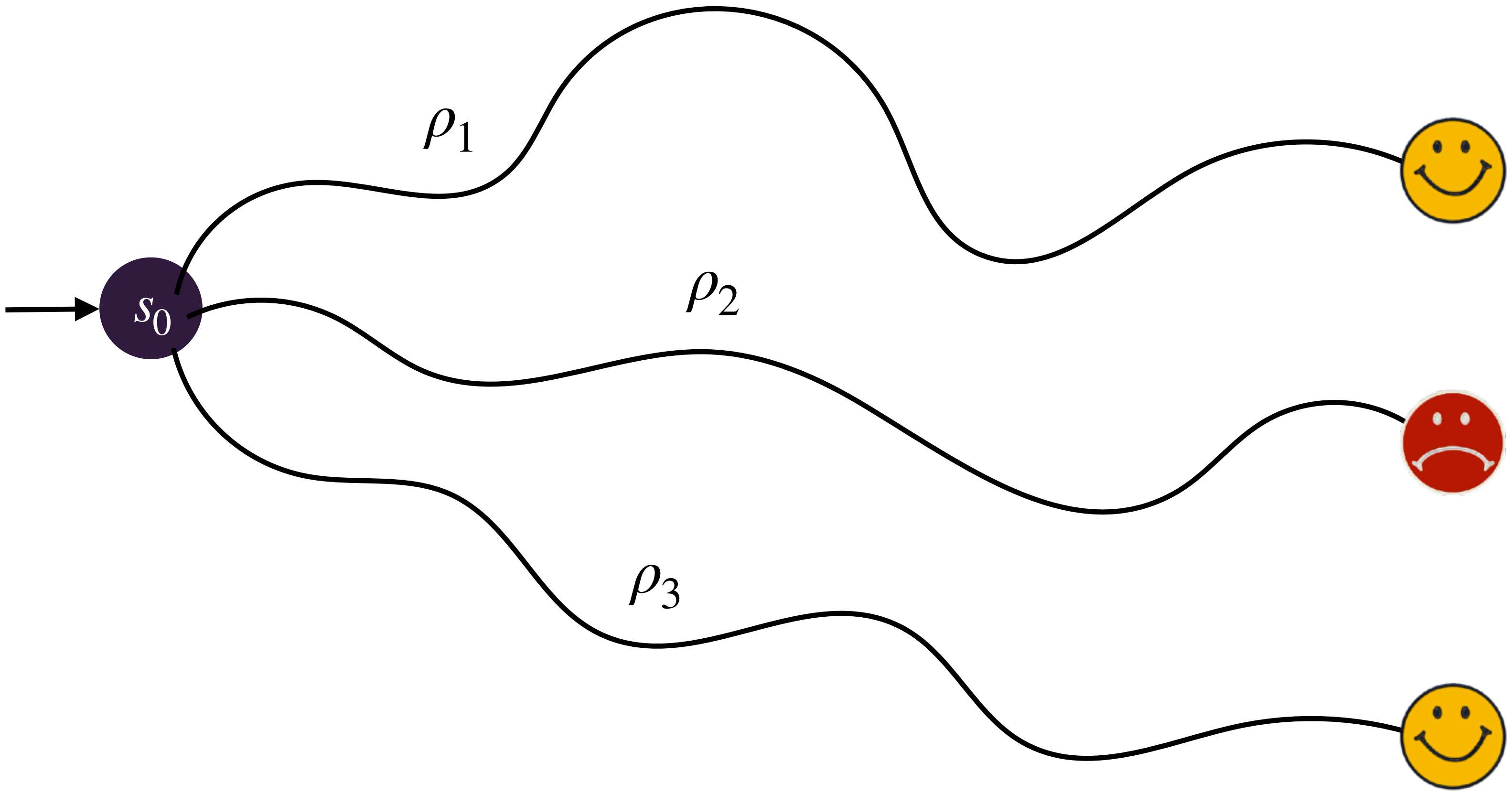

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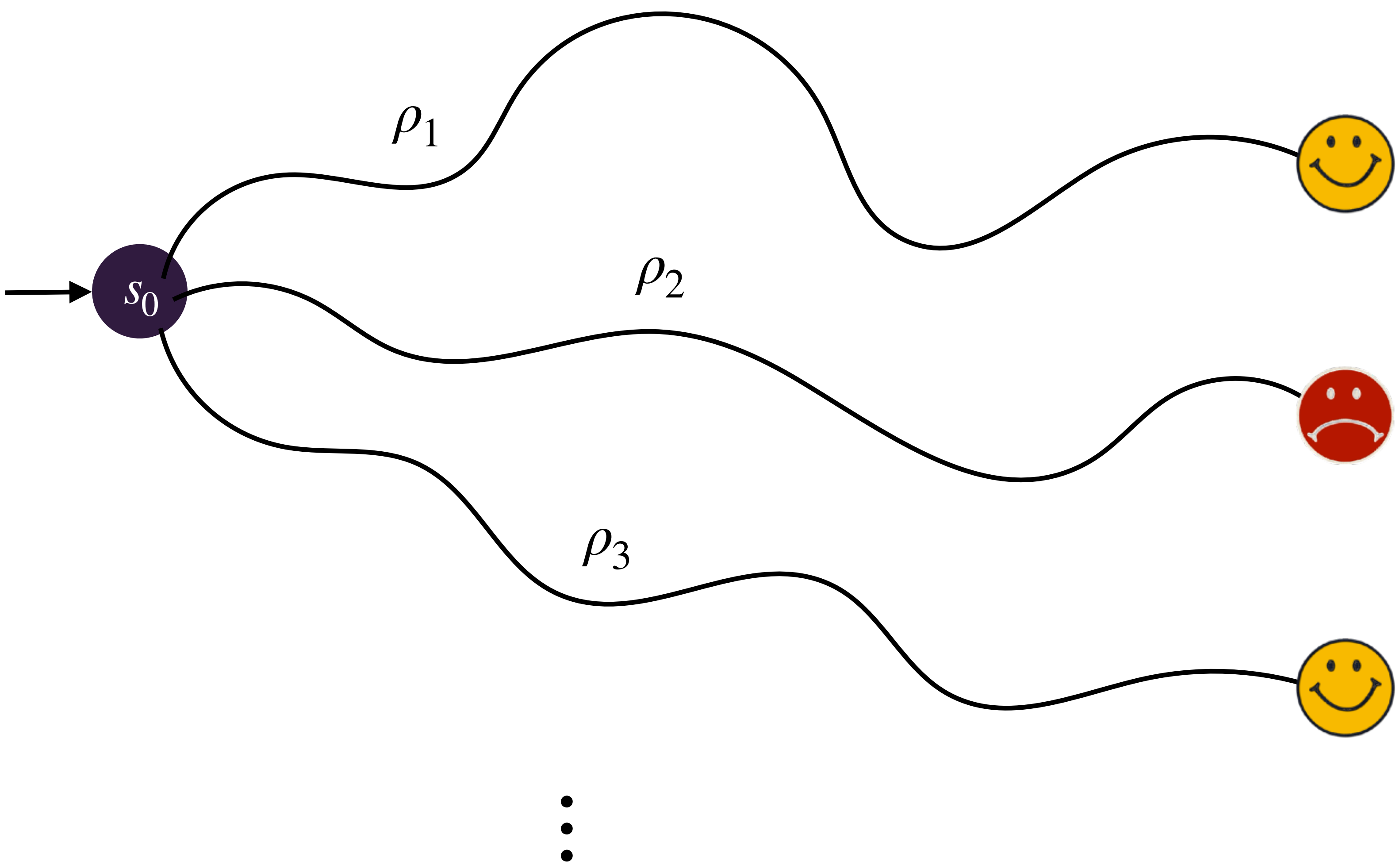

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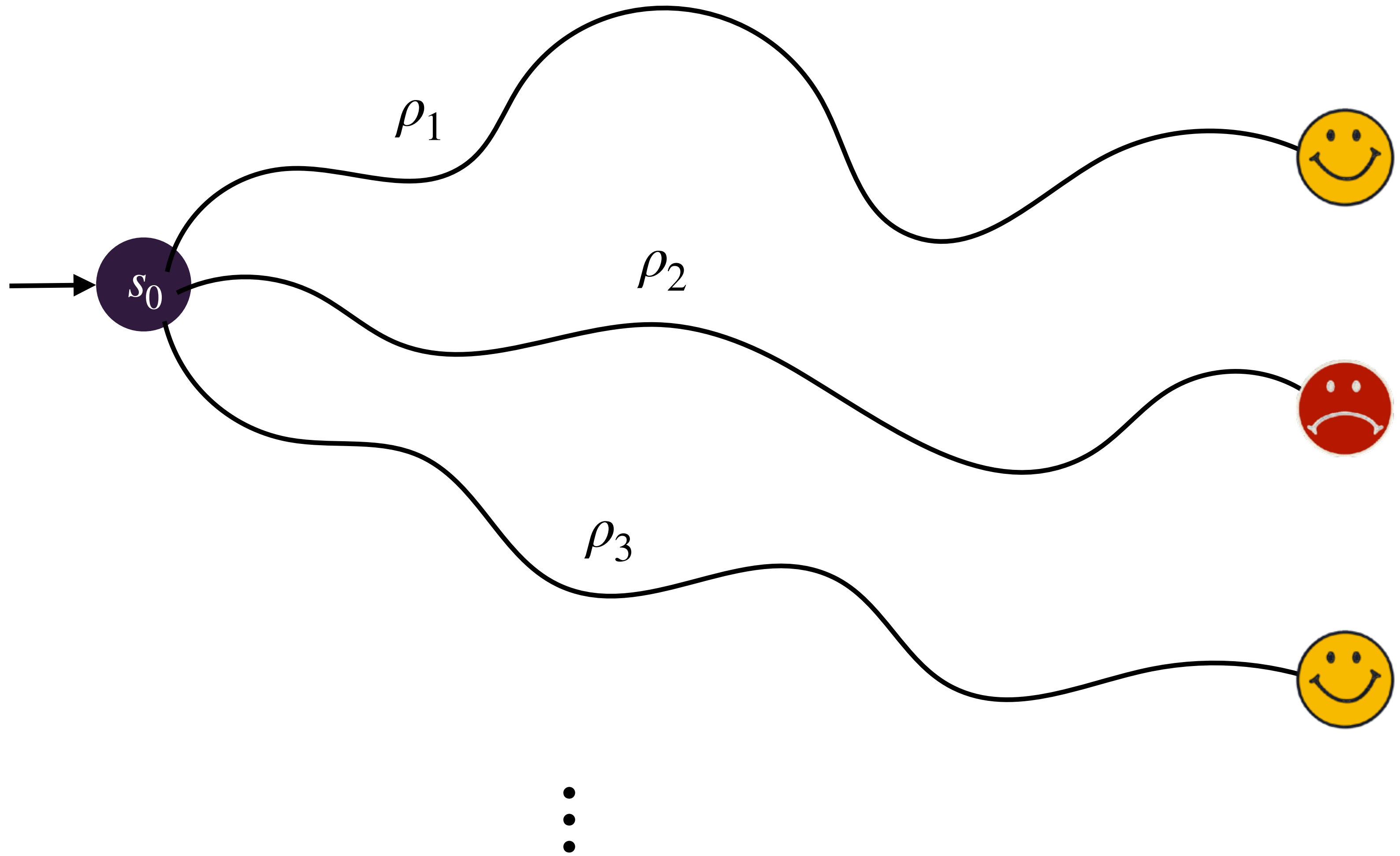

$n_1 = 1$

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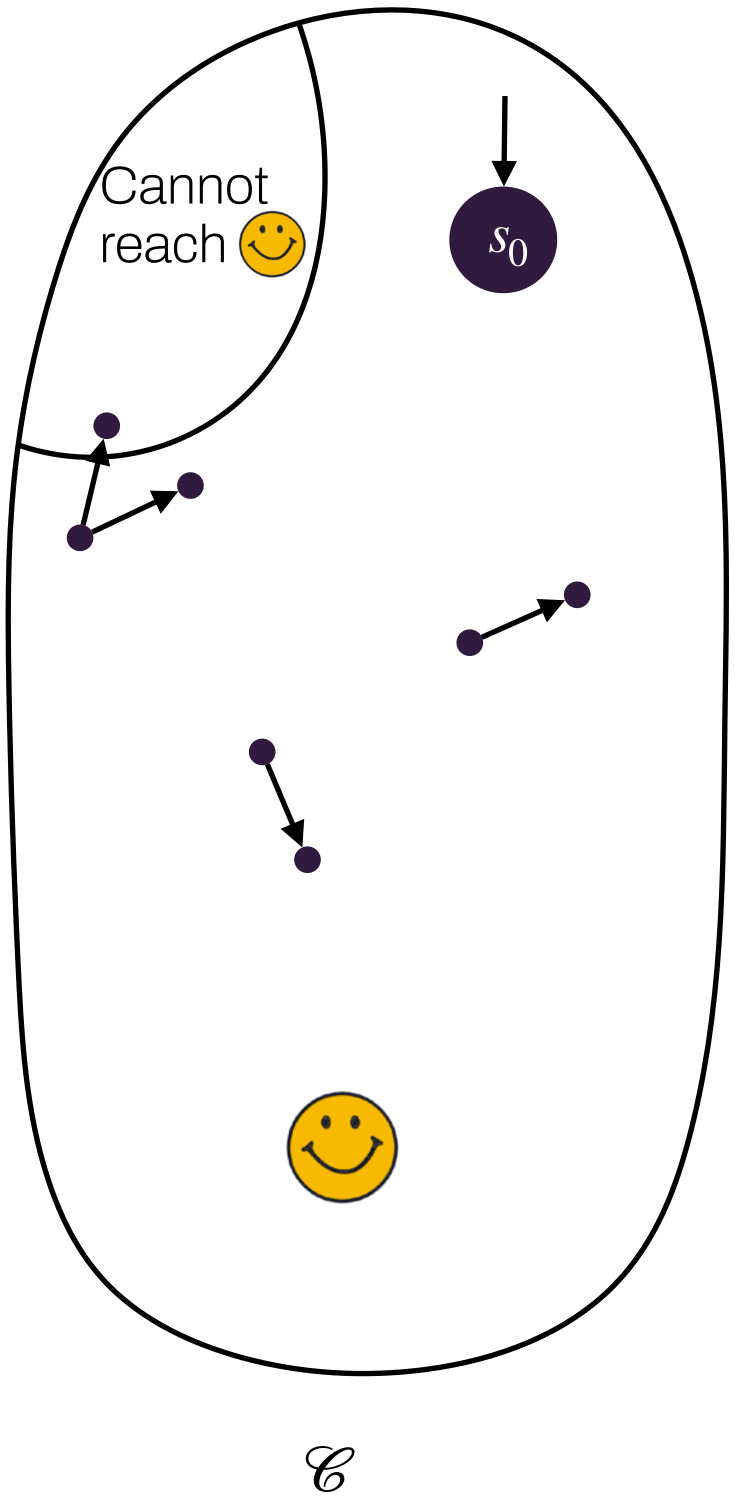
Return $\dfrac{n_N}{N}$ + some confidence interval

Termination and efficiency

Termination (To our knowledge, never expressed like this)

$\mathscr{C}$ is decisive from s_0 w.r.t. 
iff
a sampled path starting at s_0 almost-surely hits  or 

Decisiveness vs recurrence



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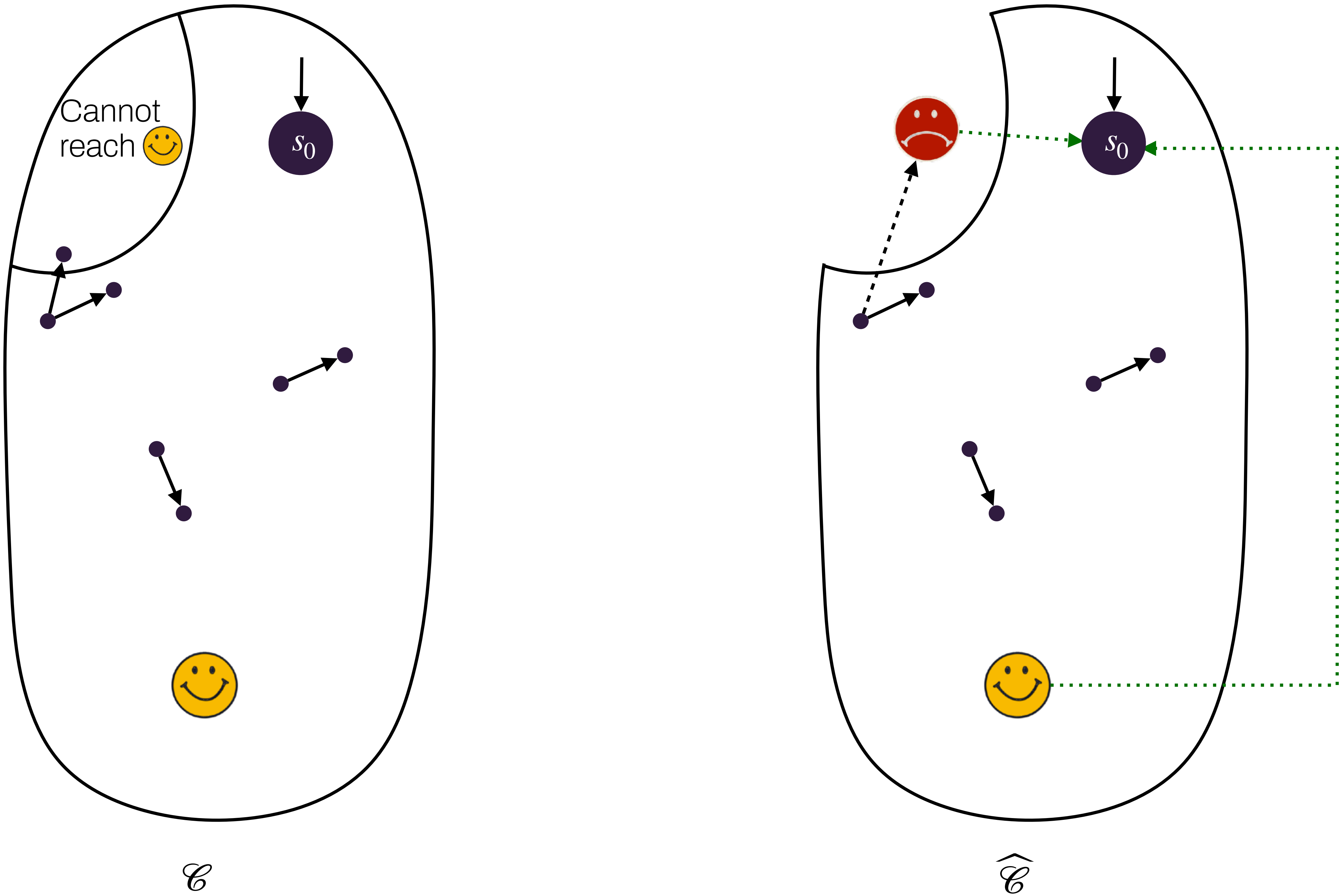

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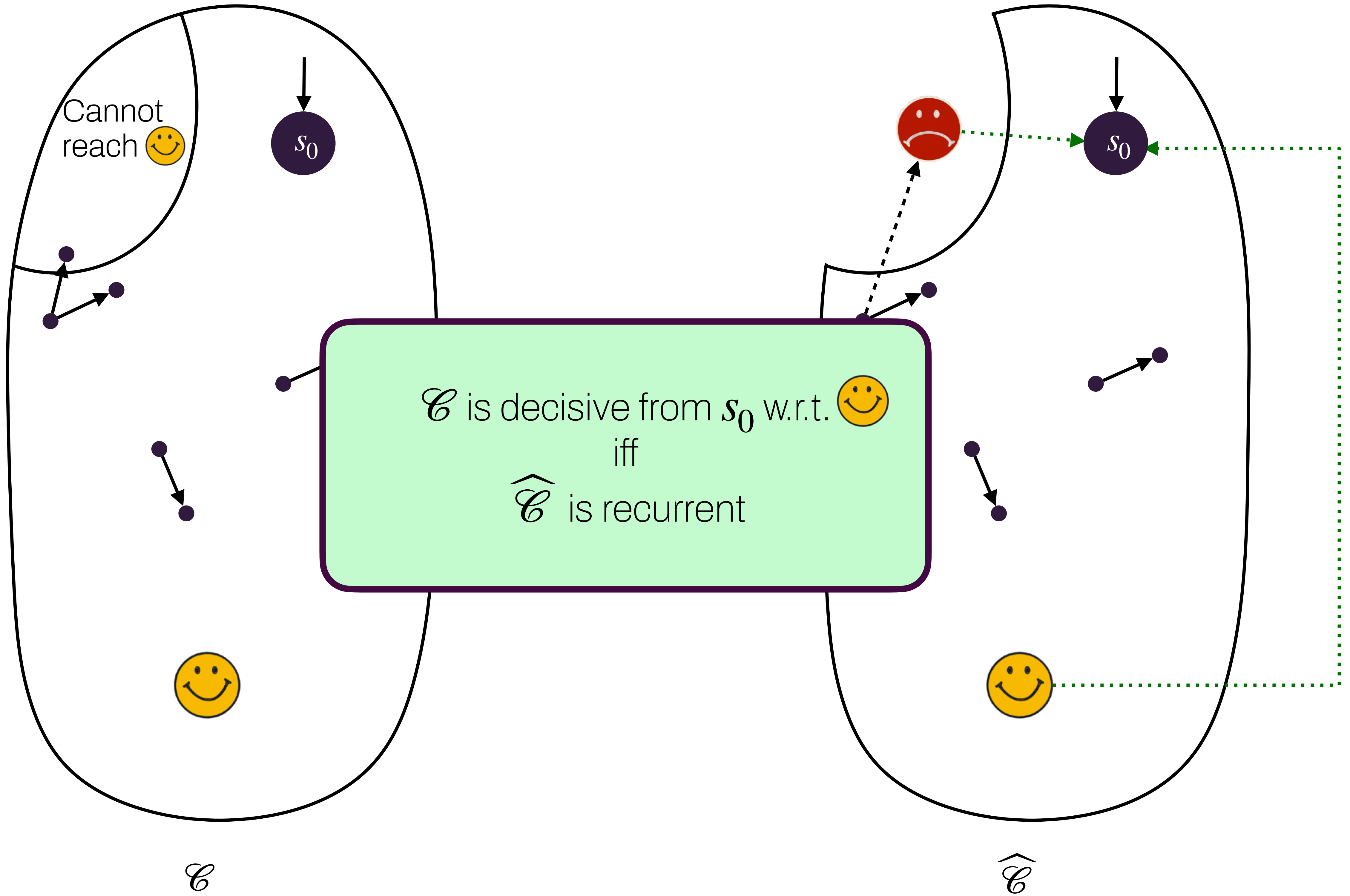

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Termination and efficiency

Termination

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$\mathscr{C}$ is decisive from s w.r.t. ☺
iff
a sampled path starting at s almost-surely hits ☺ or ☹

Efficiency of sampling

- $\mathscr{C}$ is decisive from s_0 w.r.t. ☺ iff $\widehat{\mathscr{C}}$ is recurrent

Termination and efficiency

Termination

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Efficiency of sampling

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- If $\widehat{\mathscr{C}}$ is positive recurrent, then sampling a single path in $\mathscr{C}$ will take finite time

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Efficiency of sampling

The time to sample even increases/diverges!

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- If $\widehat{\mathscr{C}}$ is positive recurrent, then sampling a single path in $\mathscr{C}$ will take finite time
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Statistical guarantees

Hoeffding's inequalities

Let $\varepsilon, \delta > 0$, let $N \geq \frac{8}{\varepsilon^2} \log\left(\frac{2}{\delta}\right)$. Then:

$$\mathbb{P}\left(\left|\frac{n_N}{N} - \mathbb{P}\left(\mathbf{F}\,\text{☺}\right)\right| \geq \frac{\varepsilon}{2}\right) \leq \delta$$

Statistical guarantees

Hoeffding's inequalities

Empirical average

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- $\left[\frac{n_N}{N} - \frac{\varepsilon}{2} ; \frac{n_N}{N} + \frac{\varepsilon}{2} \right]$: confidence interval

Statistical guarantees

Hoeffding's inequalities

Fix two parameters, the third one follows

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A slightly more general setting

- Given $L : S^+ \to \mathbb{R}$, the ☺-function $f_{L,☺}$ is $\mathbf{1}_{\mathbf{F}☺} \cdot L$
- We are interested in evaluating the quantity $\mathbb{E}(f_{L,☺})$
- If $L = \mathbf{1}_{\mathbf{F}☺}$, then $\mathbb{E}(f_{L,☺}) = \mathbb{P}(\mathbf{F}☺)$

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Empirical estimation

Let $\varepsilon, \delta > 0$ s.t. $N \geq \frac{8B^2}{\varepsilon^2} \log\left(\frac{2}{\delta}\right)$. Then:

$$\mathbb{P}\left(\left|\frac{f_N}{N} - \mathbb{E}(f_{L,☺})\right| \geq \frac{\varepsilon}{2}\right) \leq \delta$$

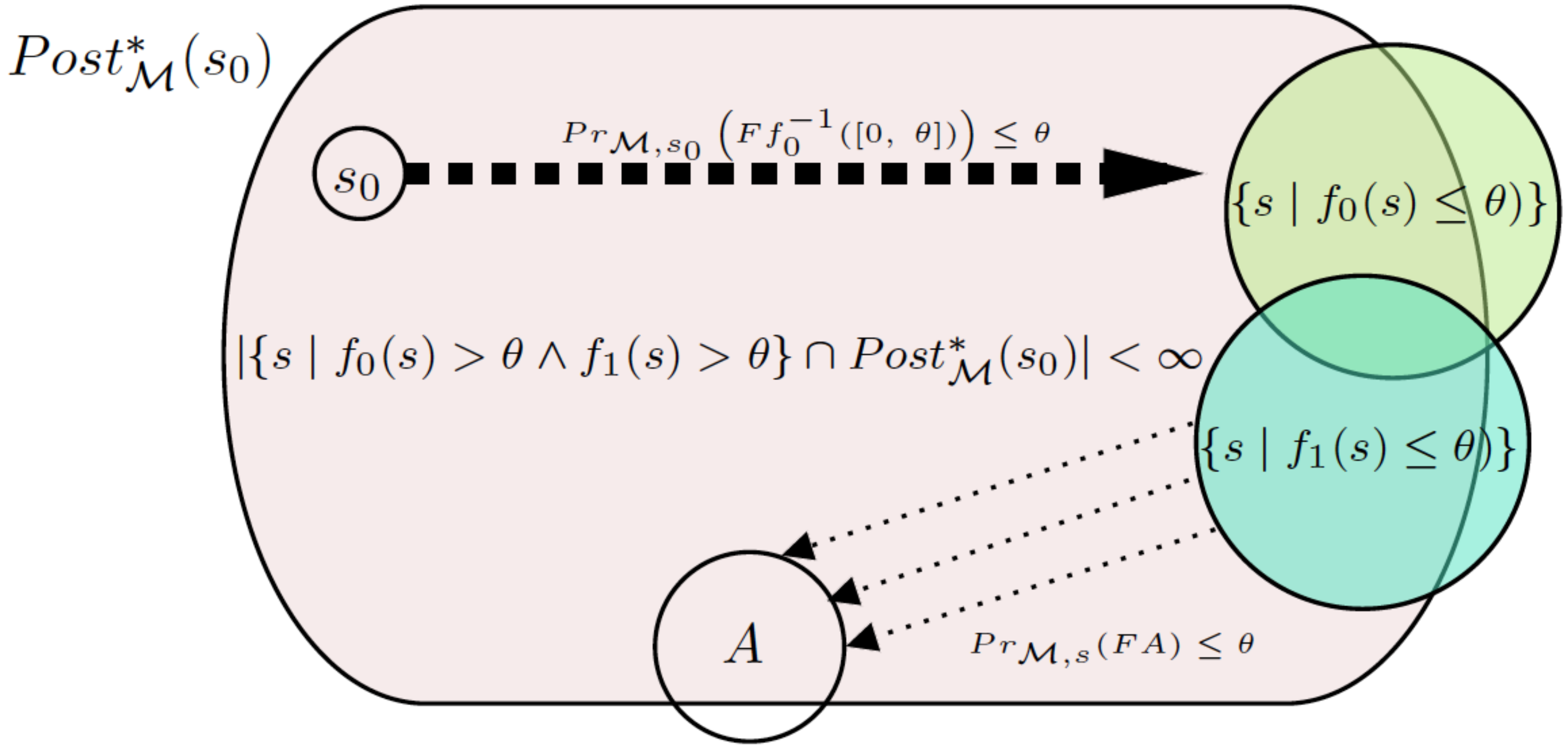
What can we do for
non-decisive Markov chains??

Another numerical generic approach

Divergent Markov Chains

A Markov chain $\mathcal{M}$ is *divergent* w.r.t. s_0 and A if there exist two computable functions f_0 and f_1 from S to $\mathbb{R}_{\geq 0}$ such that:

1. For all $0 < \theta < 1$, $\mathbf{Pr}_{\mathcal{M},s_0}(\mathbf{F} f_0^{-1}([0,\theta])) \leq \theta$;
2. For all $s \in S$, $\mathbf{Pr}_{\mathcal{M},s}(\mathbf{F} A) \leq f_1(s)$;
3. For all $0 < \theta < 1$, $\{s \mid f_0(s) > \theta \wedge f_1(s) > \theta\} \cap Post^*_{\mathcal{M}}(s_0)$ is finite.



[FHY23b] A. Finkel, S. Haddad, L. Ye. Introducing divergence for infinite probabilistic models (RP'23)

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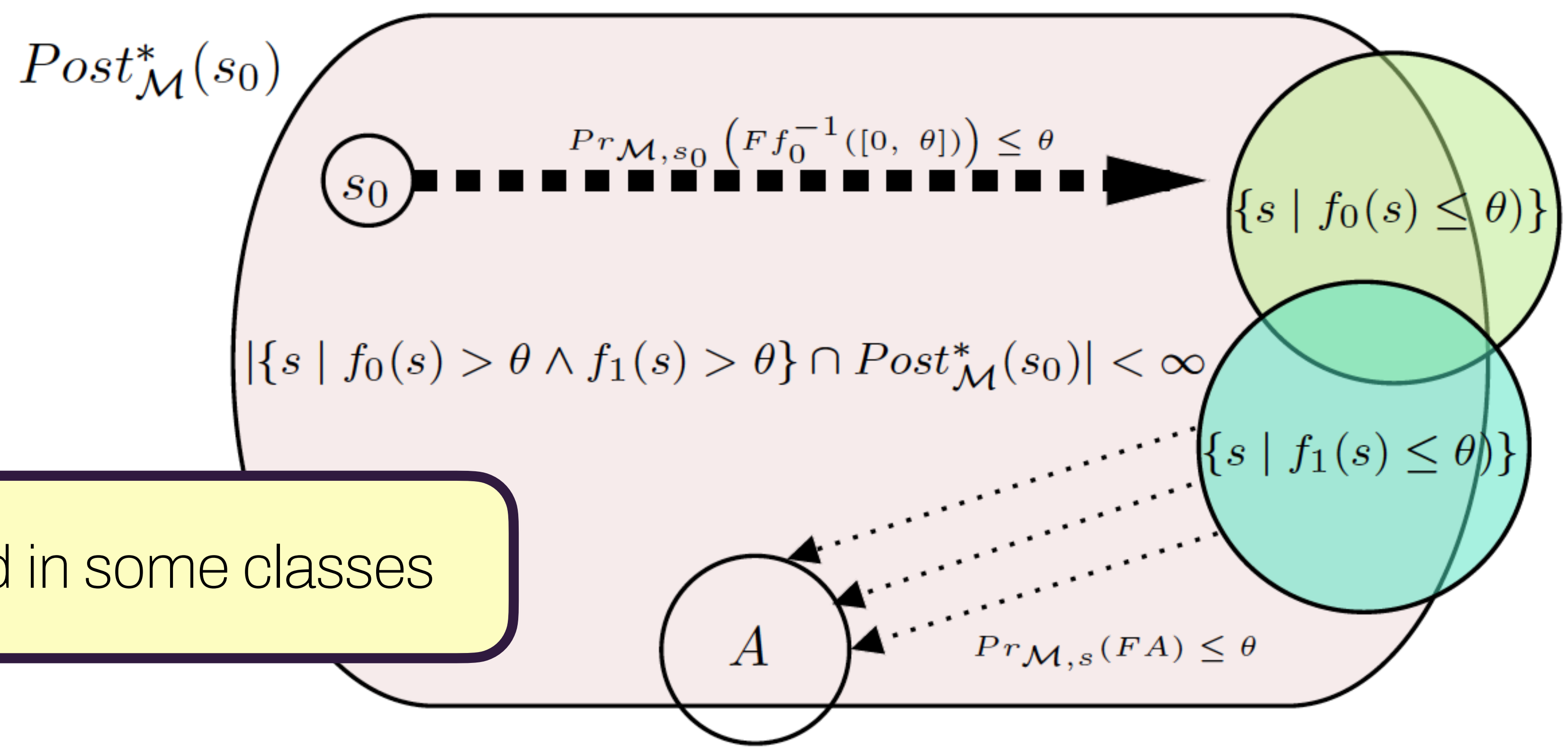

Can be decided in some classes

[FHY23b] A. Finkel, S. Haddad, L. Ye. Introducing divergence for infinite probabilistic models (RP'23)

Importance sampling for rare events evaluation

- Issue: rare events in $\mathscr{C}$

Rare-Event Problem for Statistical Model Checking

Problem Statement

- We want to estimate the probability of a rare event e occurring with probability close to 10^{-15}.
- We want a *confidence level* of 0.99.
- We are able to compute 10^9 trajectories.

Possible Outcomes

Number of occurrences of e	Probability	Confidence interval
0	$\approx 1 - 10^{-6}$	$[0, 7.03 \cdot 10^{-9}]$
1	$\leq 10^{-6}$	$[6.83 \cdot 10^{-10}, 1.69 \cdot 10^{-9}]$
$n > 1$	$\leq 10^{-12}$	$> 6.83 \cdot 10^{-10}$

[Bar14] B. Barbot. Acceleration for statistical model checking (PhD thesis)
[BHP12] B. Barbot, S. Haddad, C. Picaronny. Coupling and Importance Sampling for Statistical Model Checking (TACAS'12)

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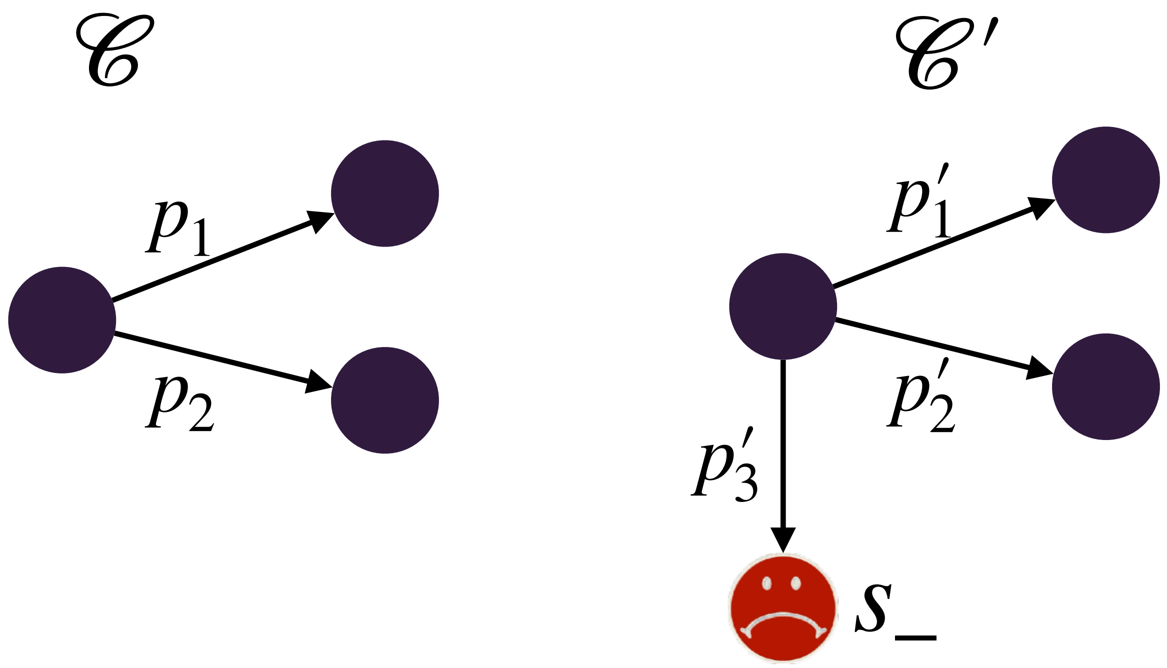

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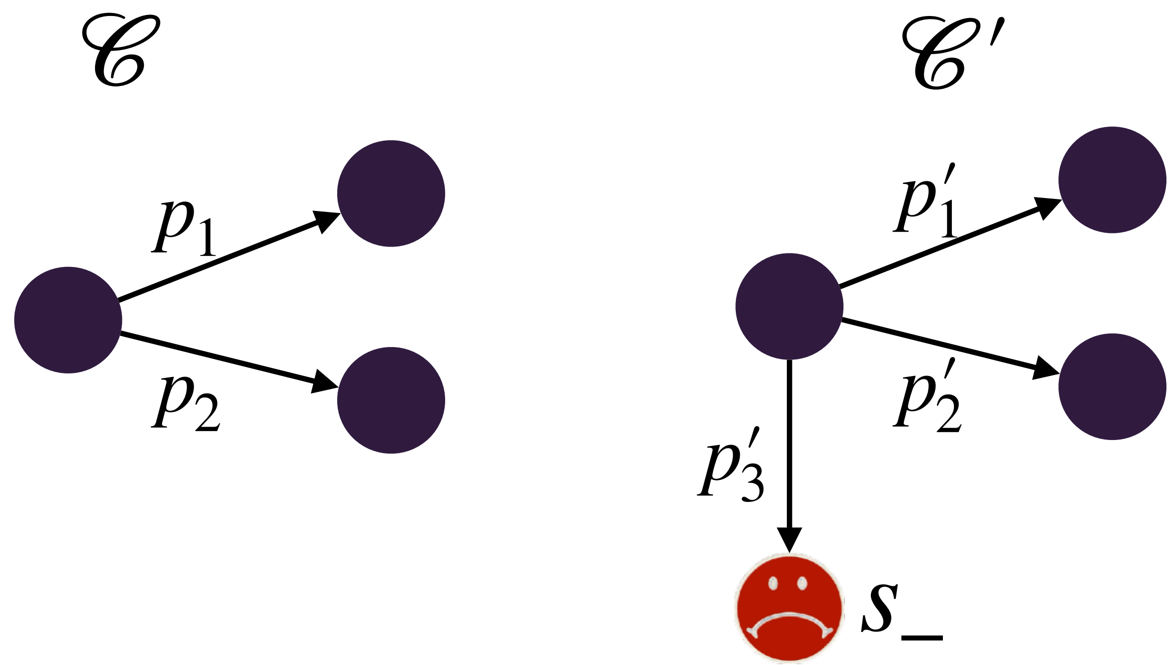
Likelihood and biased function

$$\gamma(\rho) = \begin{cases} \dfrac{P(\rho)}{P'(\rho)} & \text{if } \rho \text{ ends in} \\ 0 & \text{otherwise} \end{cases}$$

$$L' = L \cdot \gamma$$



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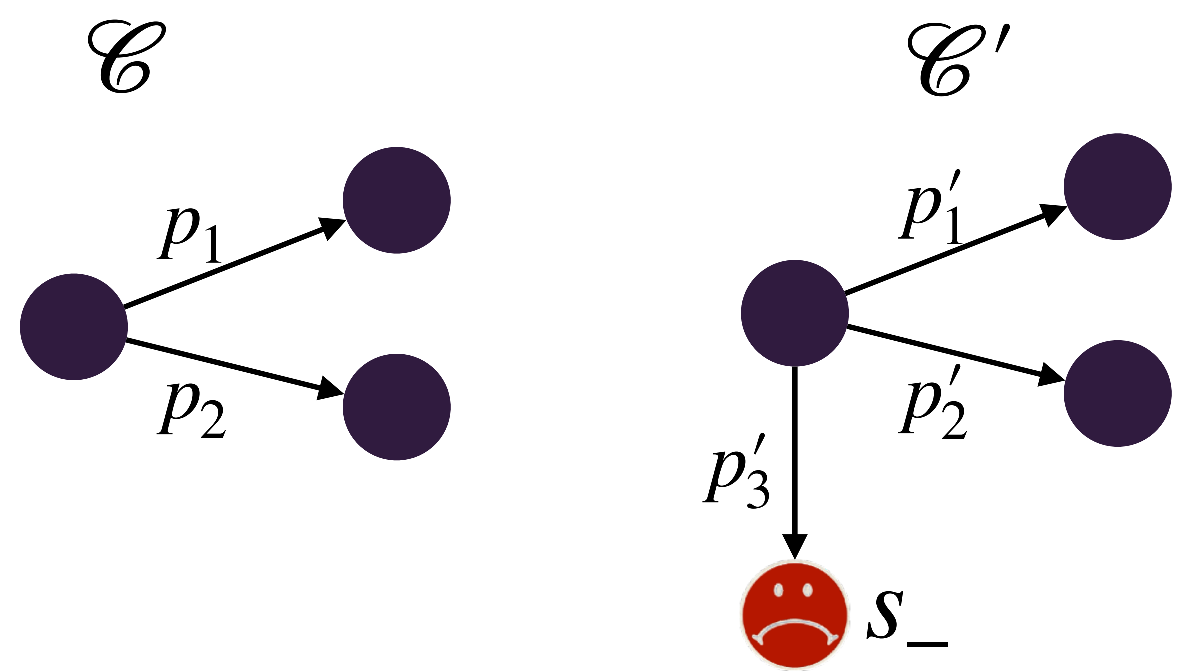

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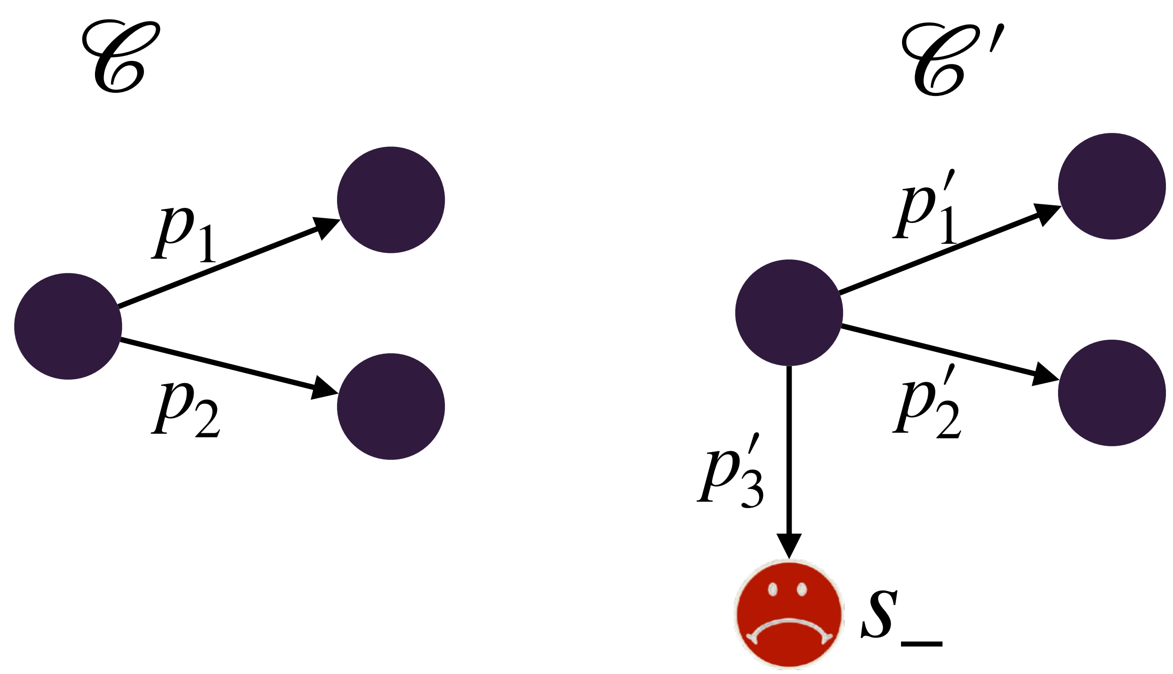

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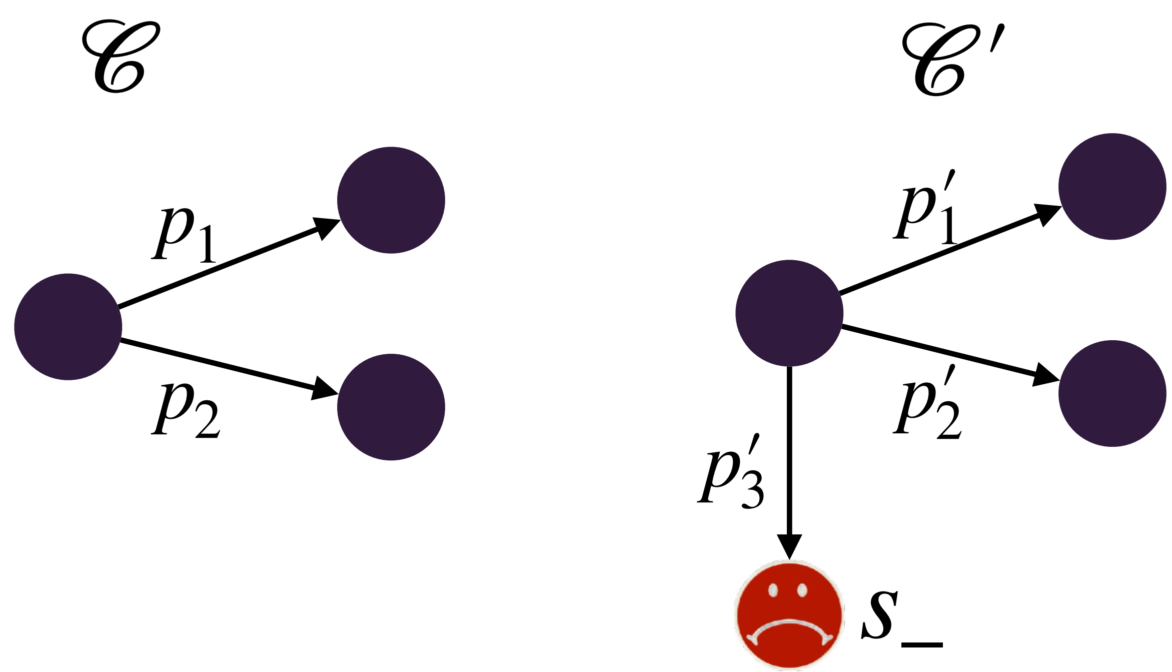

+ setting giving statistical guarantees

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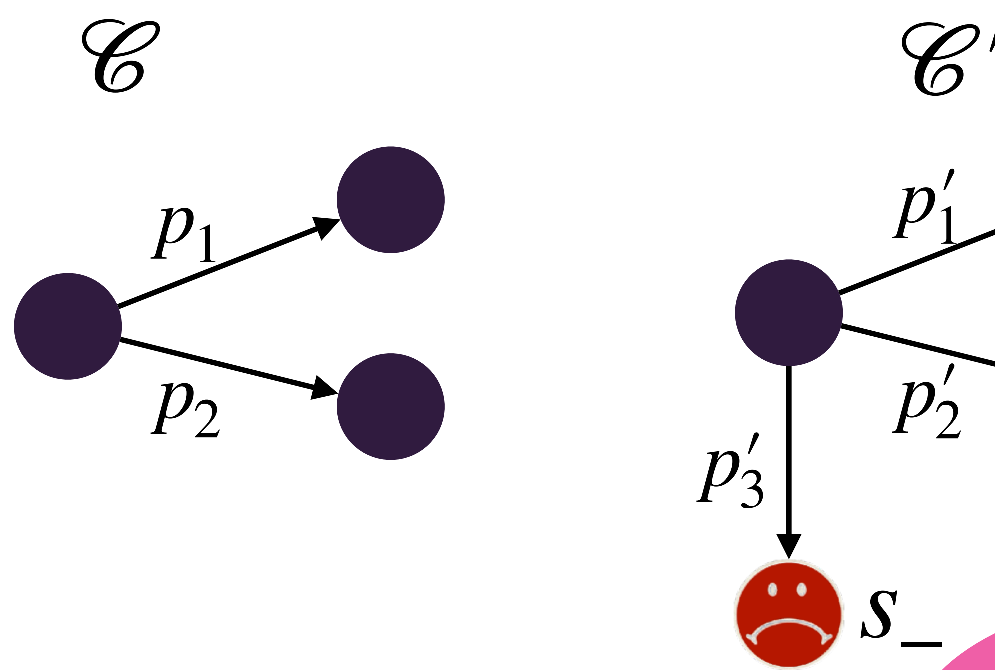

Before that, only estimators!!

+ setting giving statistical guarantees

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We propose to use the importance sampling approach to analyze some non-decisive DTMCs!

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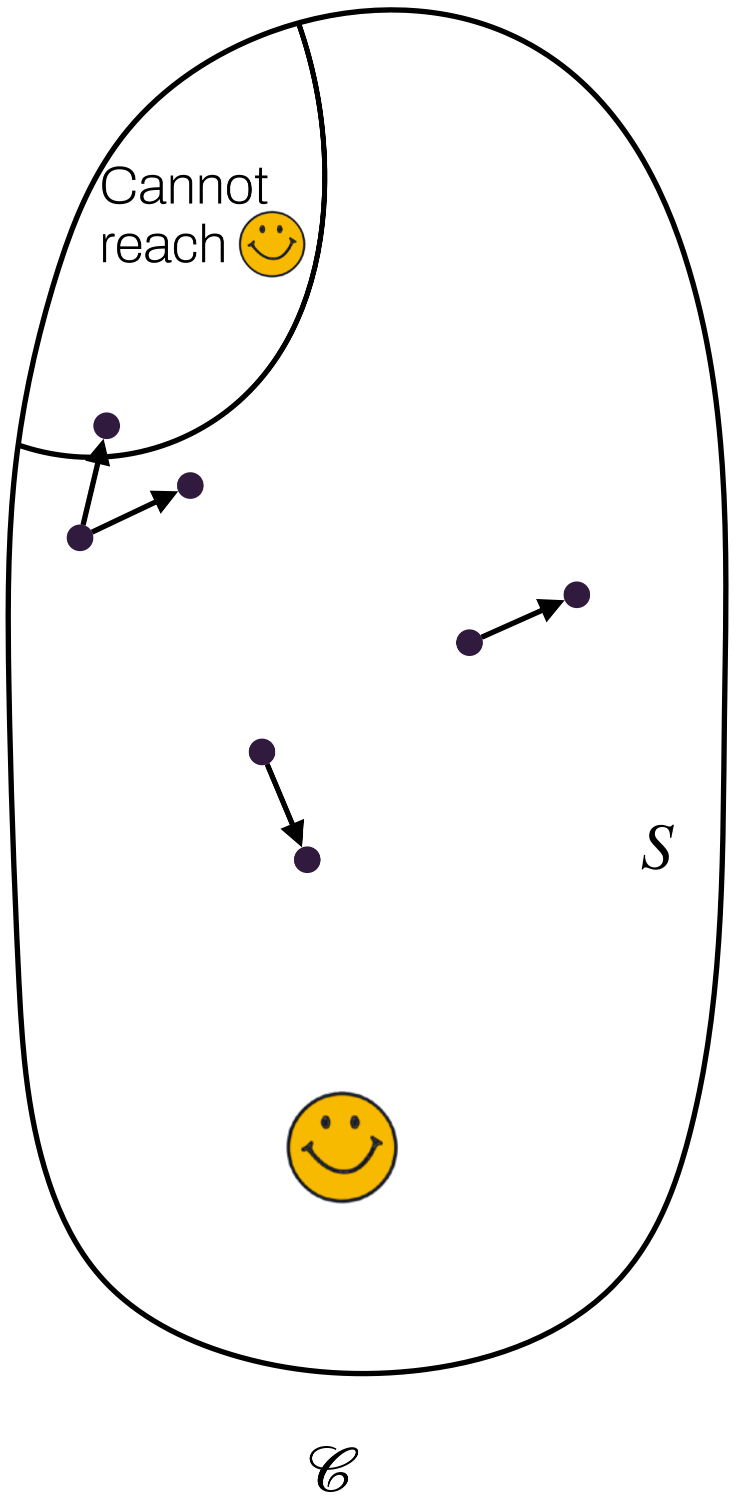
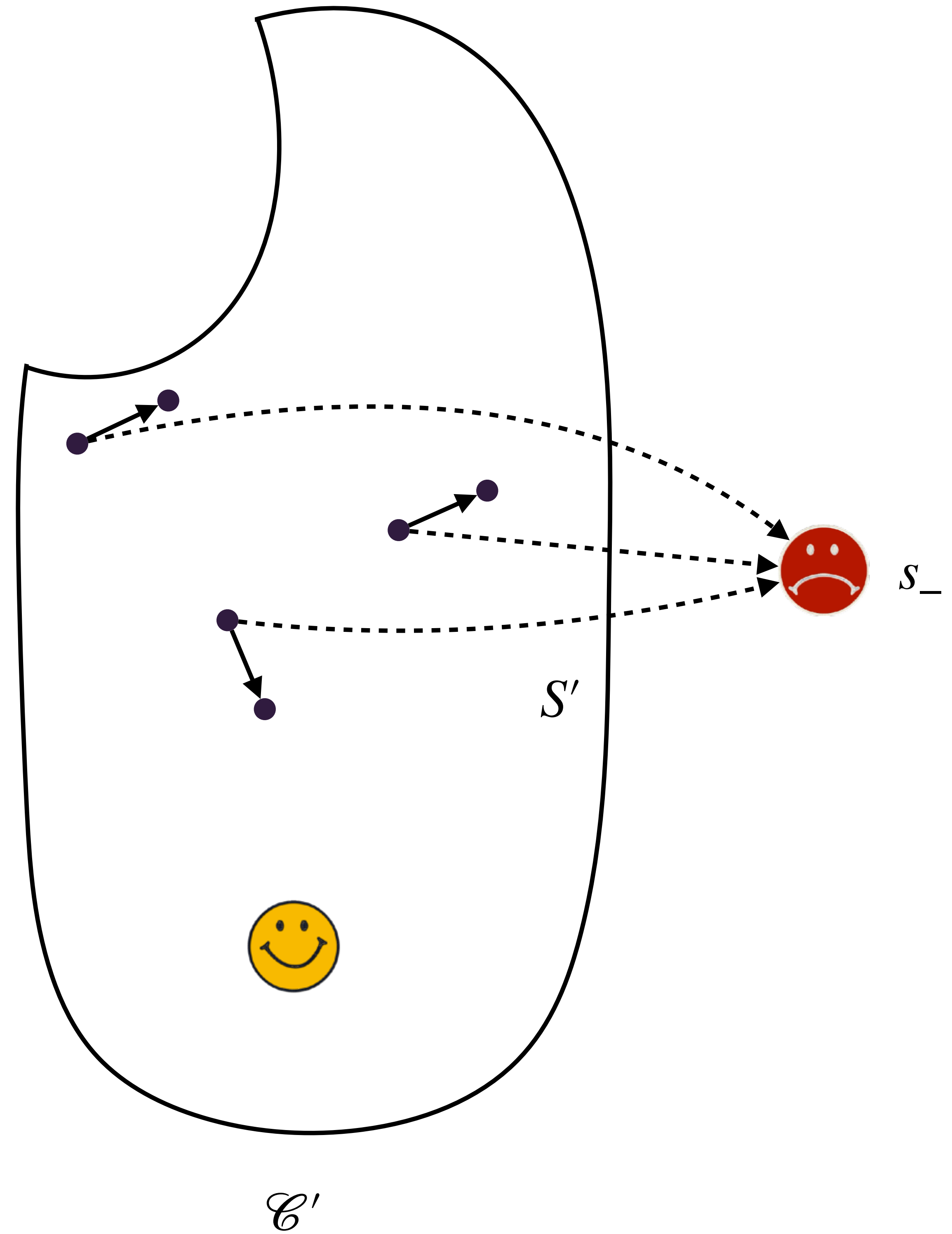
First time that importance sampling is used not to accelerate the analysis, but to enable the analysis

Biased Markov chain





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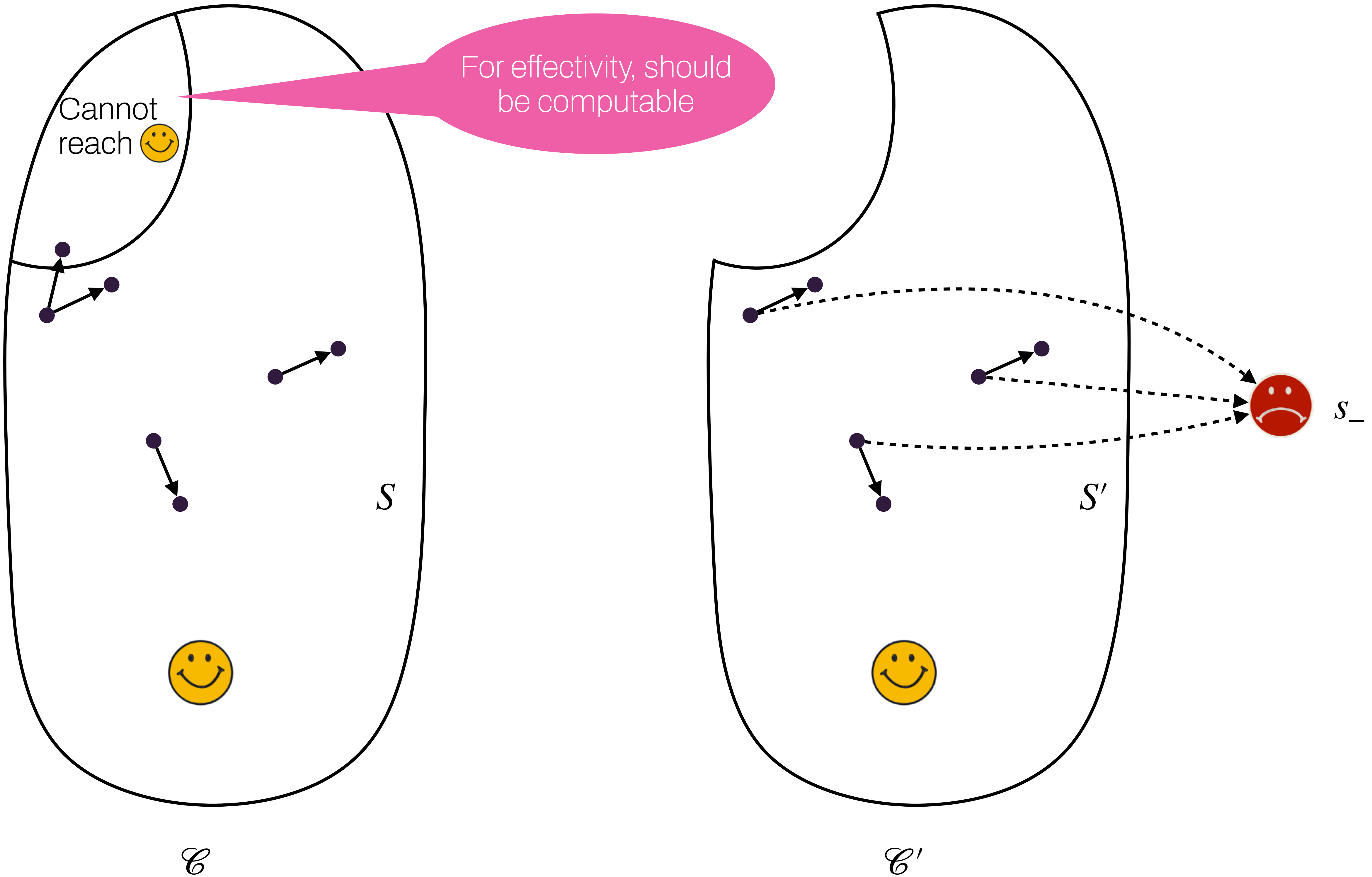

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$$\gamma(\rho) = \begin{cases} \frac{P(\rho)}{P'(\rho)} & \text{if } \rho \text{ ends in ☺} \\ 0 & \text{otherwise} \end{cases}$$

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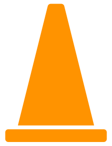
  - L' can be unbounded even if L is bounded 

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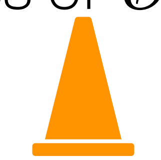
  - L' can be unbounded even if L is bounded 
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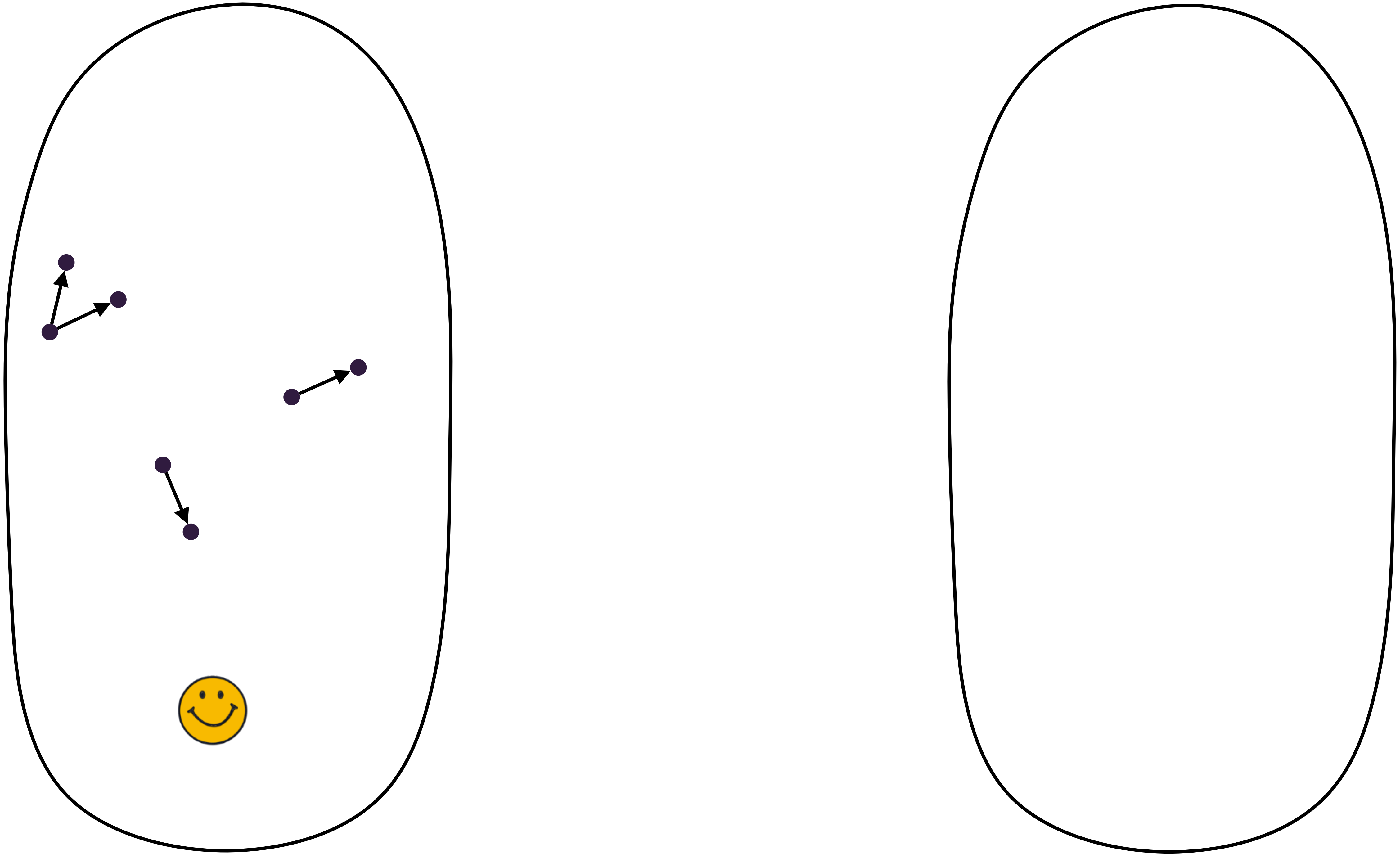
Our approach



$\mathscr{C}$

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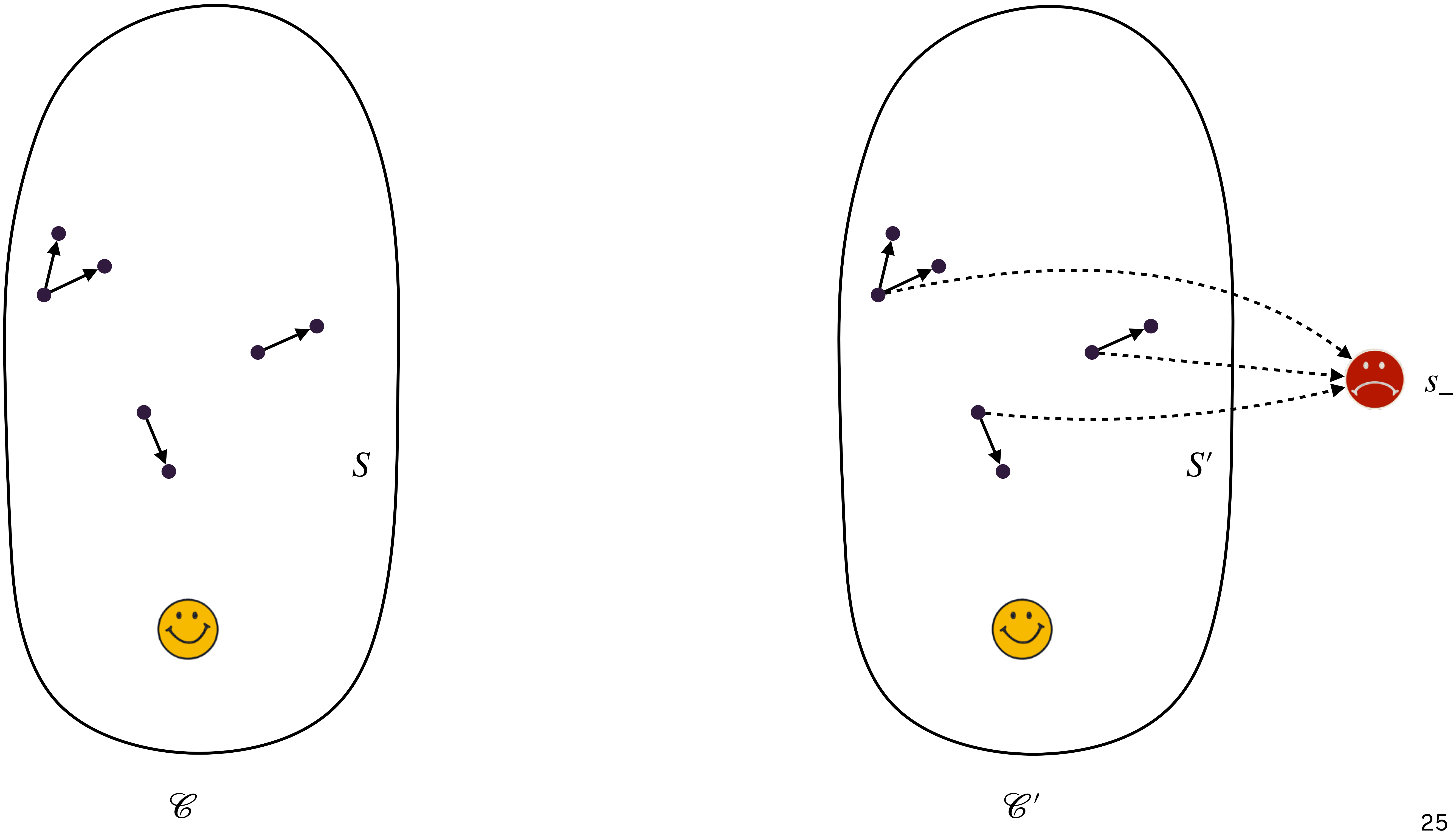

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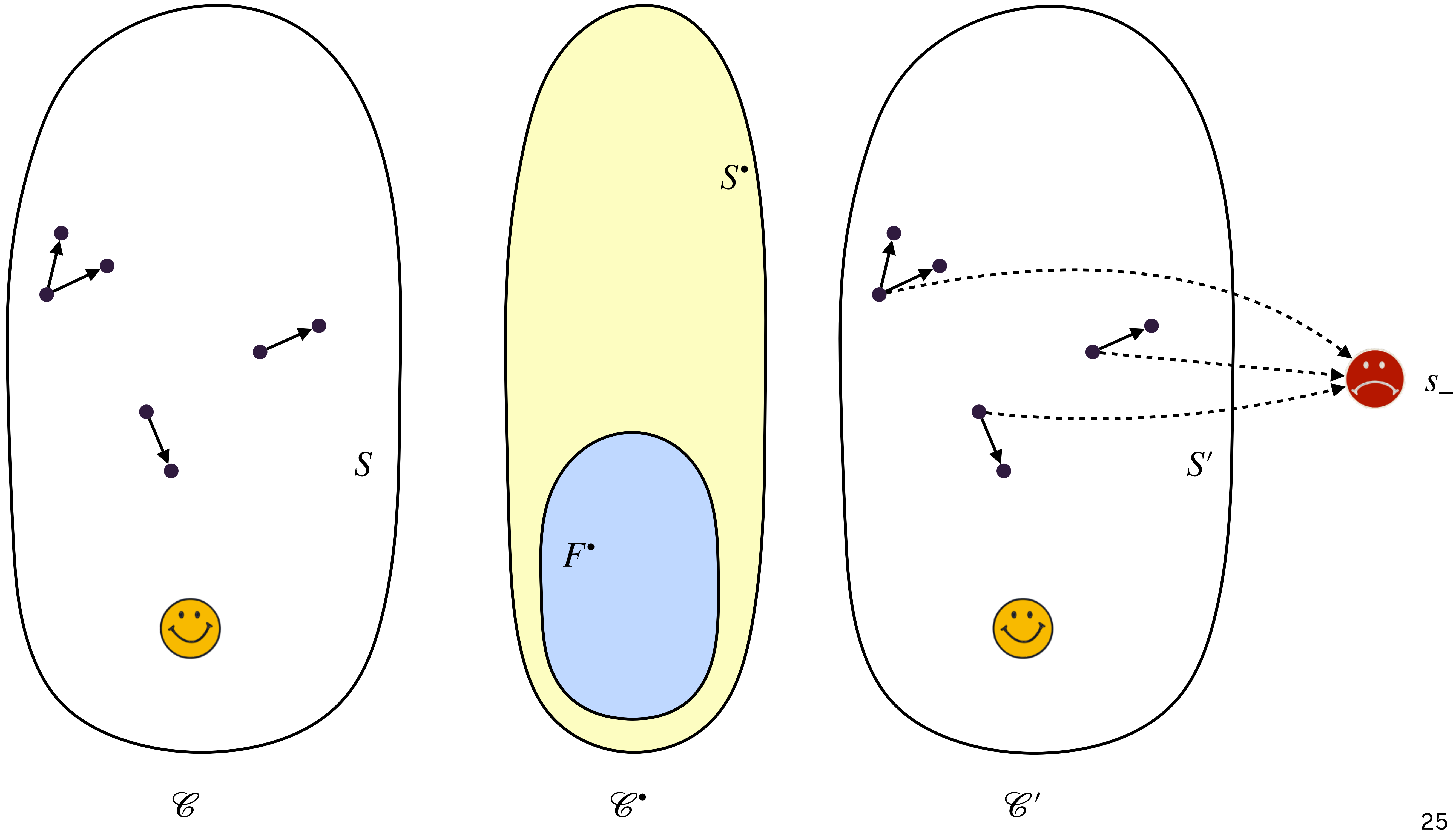

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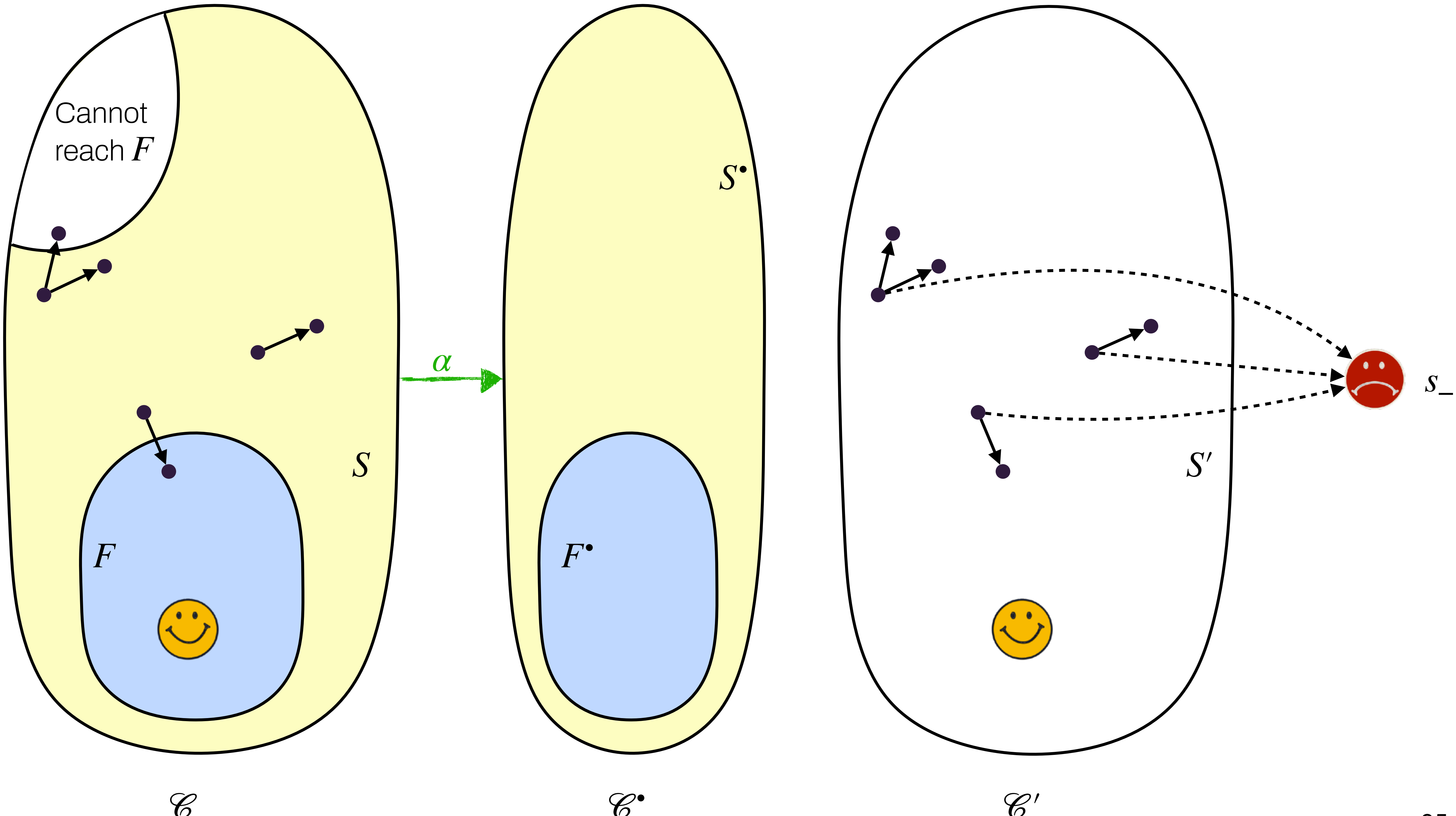

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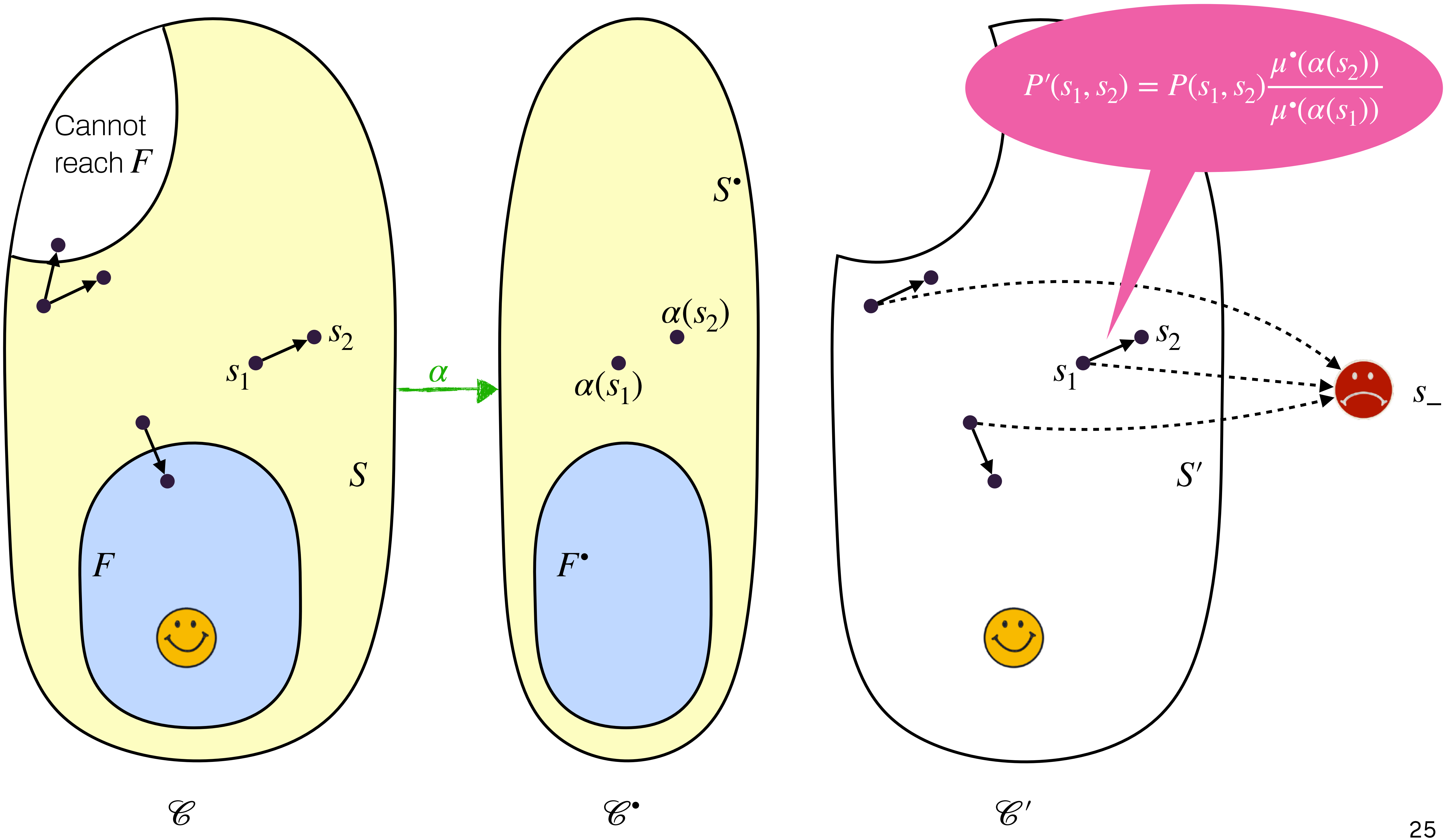

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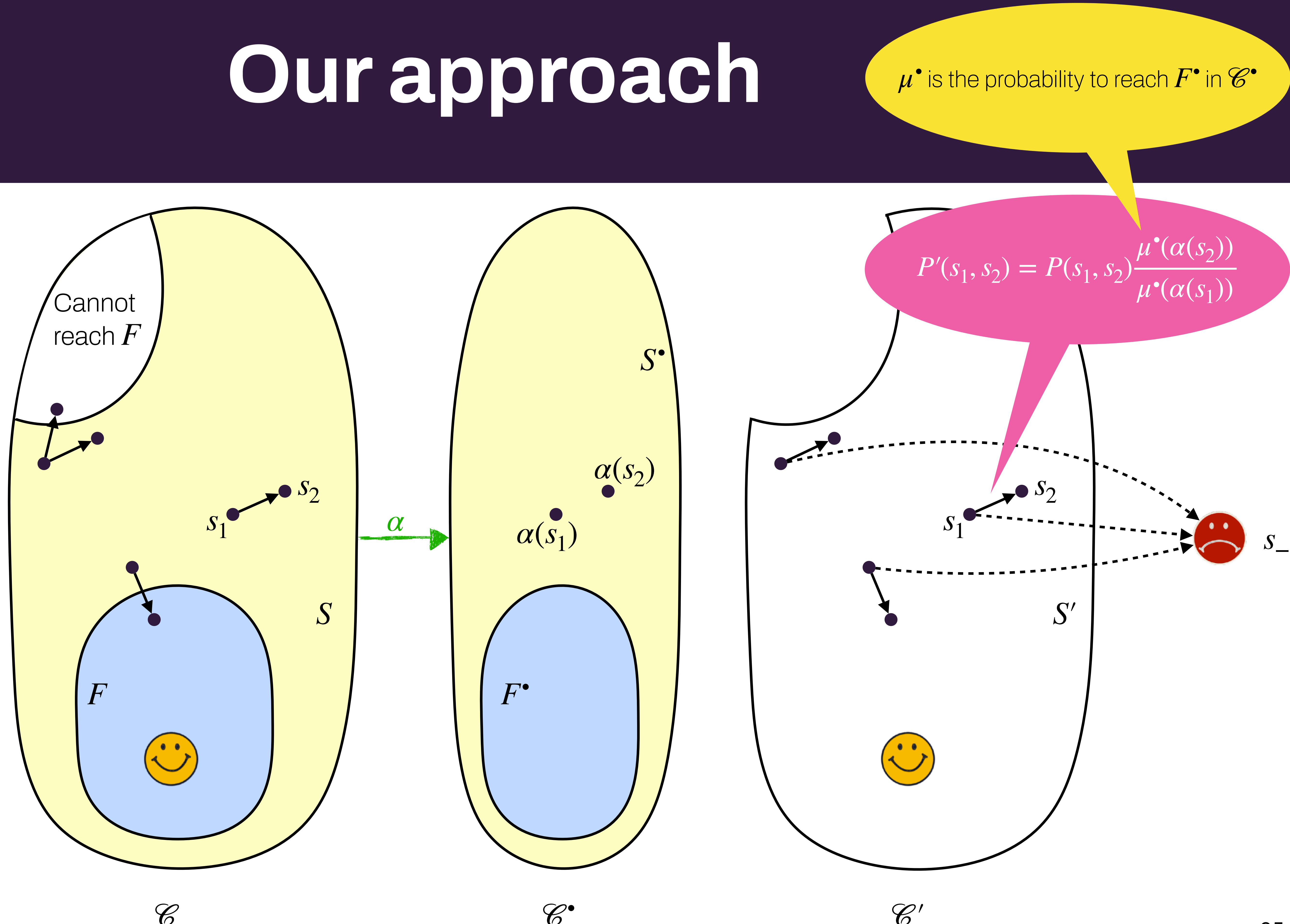

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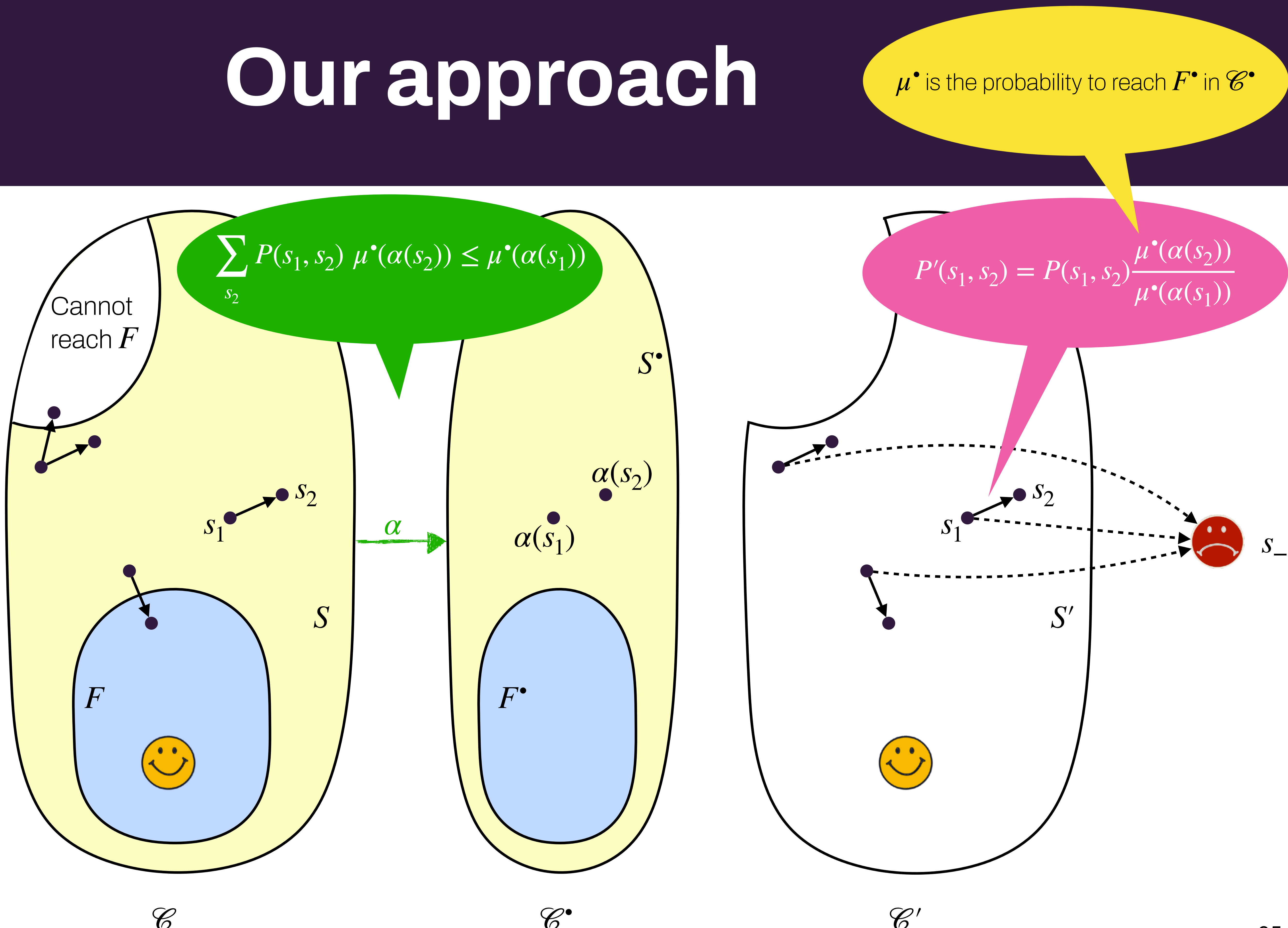

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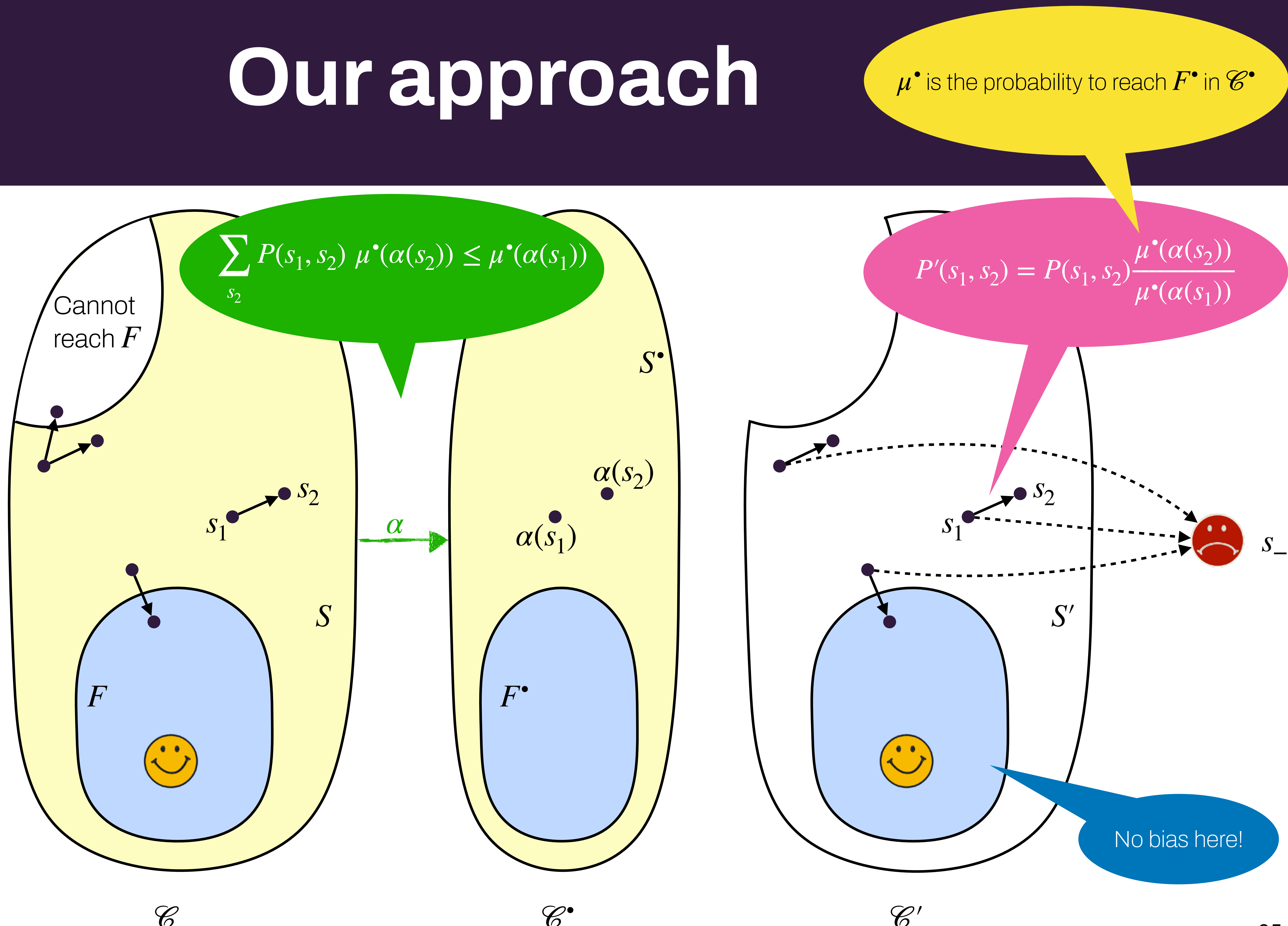

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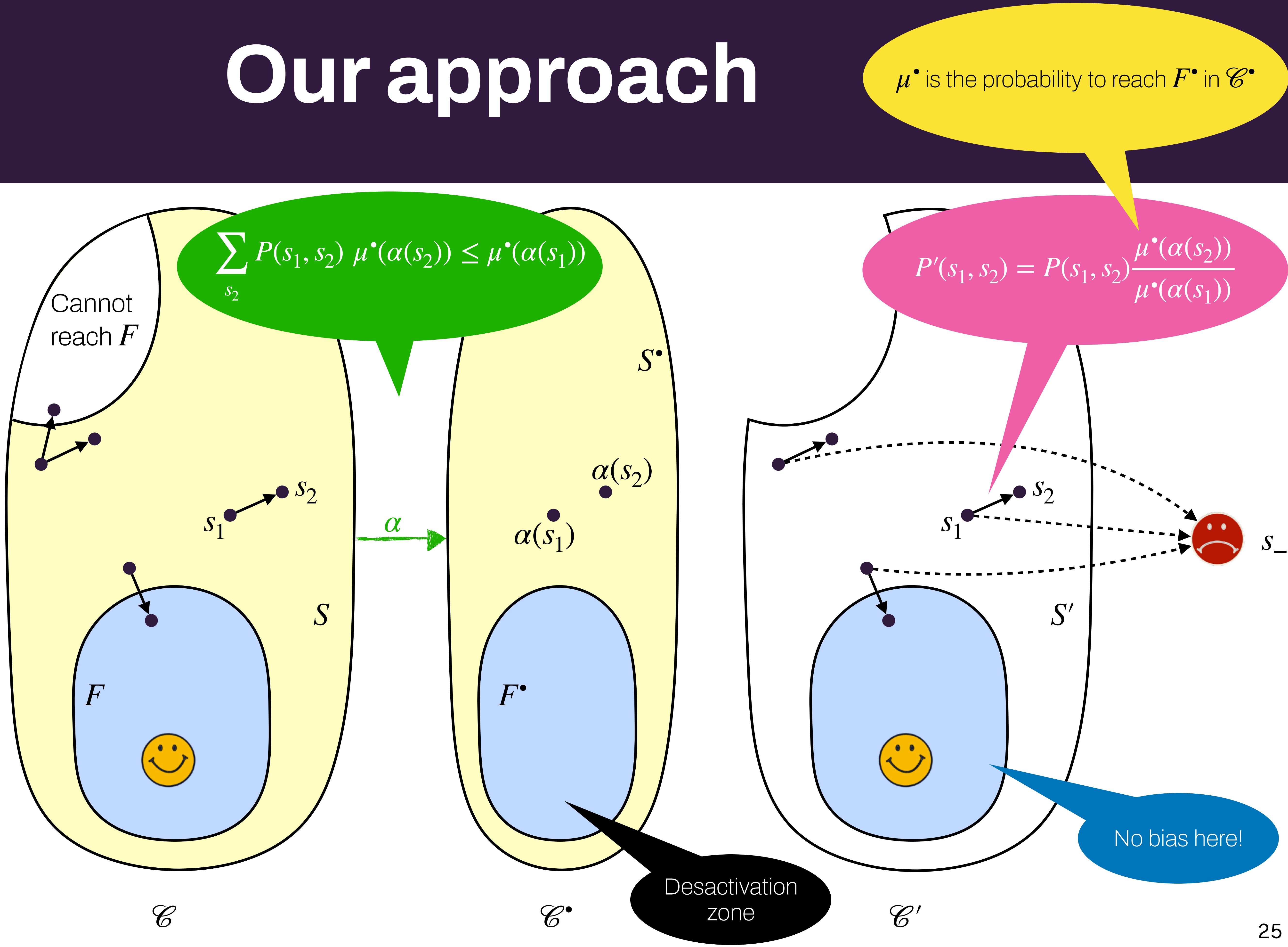

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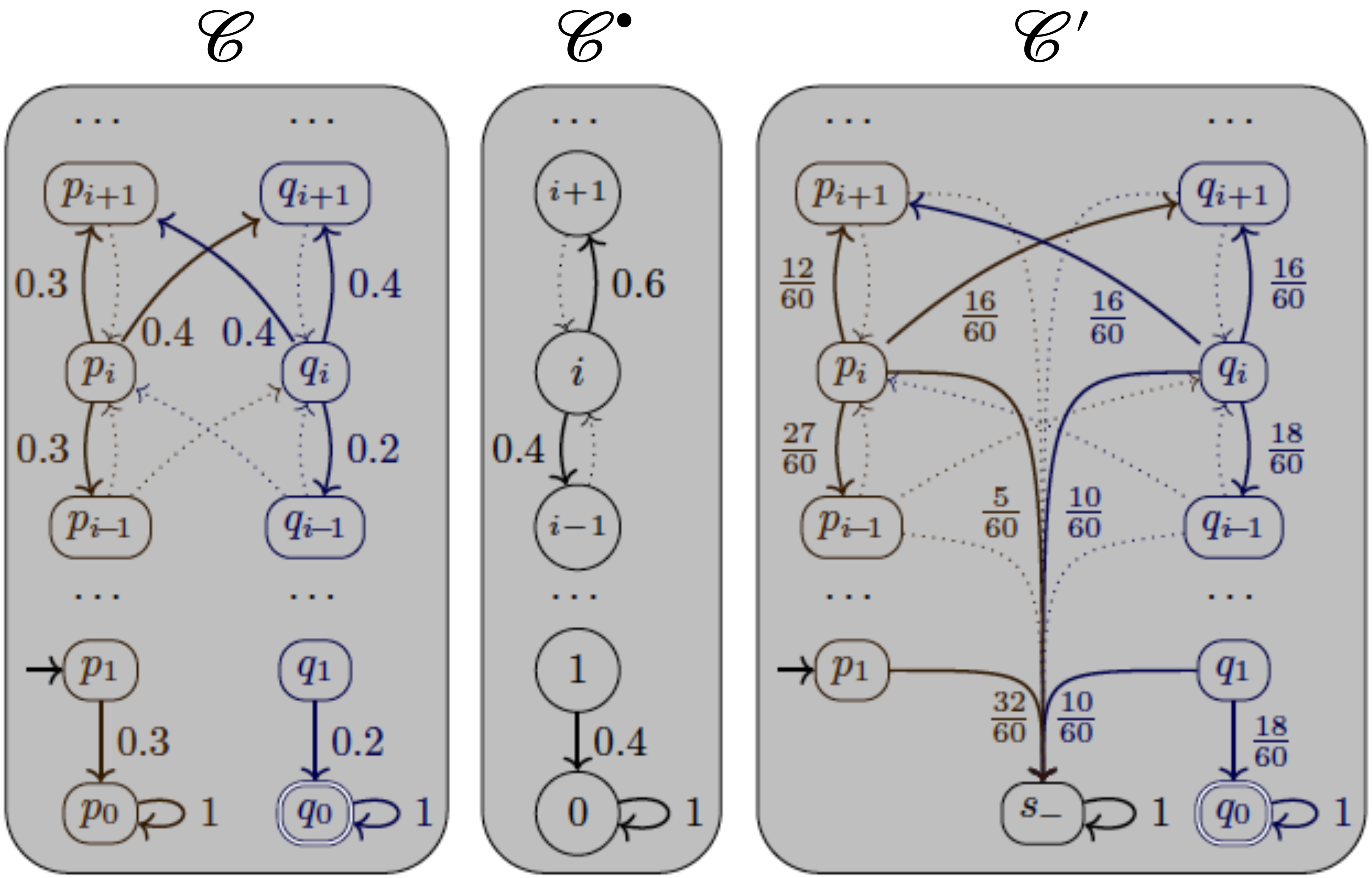
  - To compute $\mu^{\bullet}(\,\cdot\,)$ (useful in two places: to sample paths and to compute the final value when hitting $\smiley$)

Role of F

- Standard approach for importance sampling: no set F
  (F coincides with 😊)
- Will be useful to adjust the properties satisfied by the abstraction to be correct
  - Requirement will be « outside F »
  - For instance, congestion of systems

Example



$F = \{ ☺ \} = \{q_0\}$

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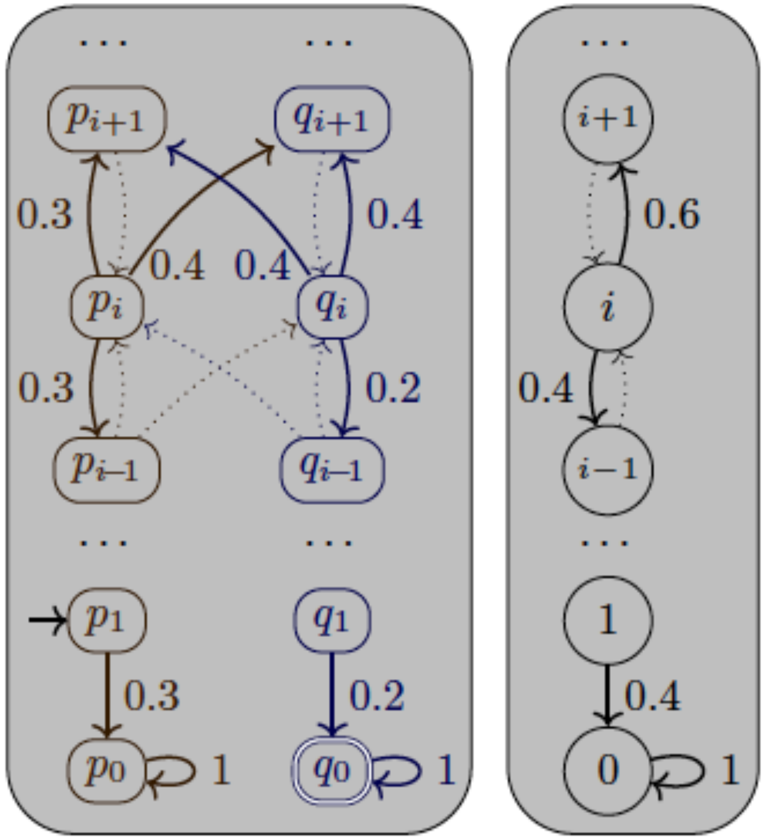

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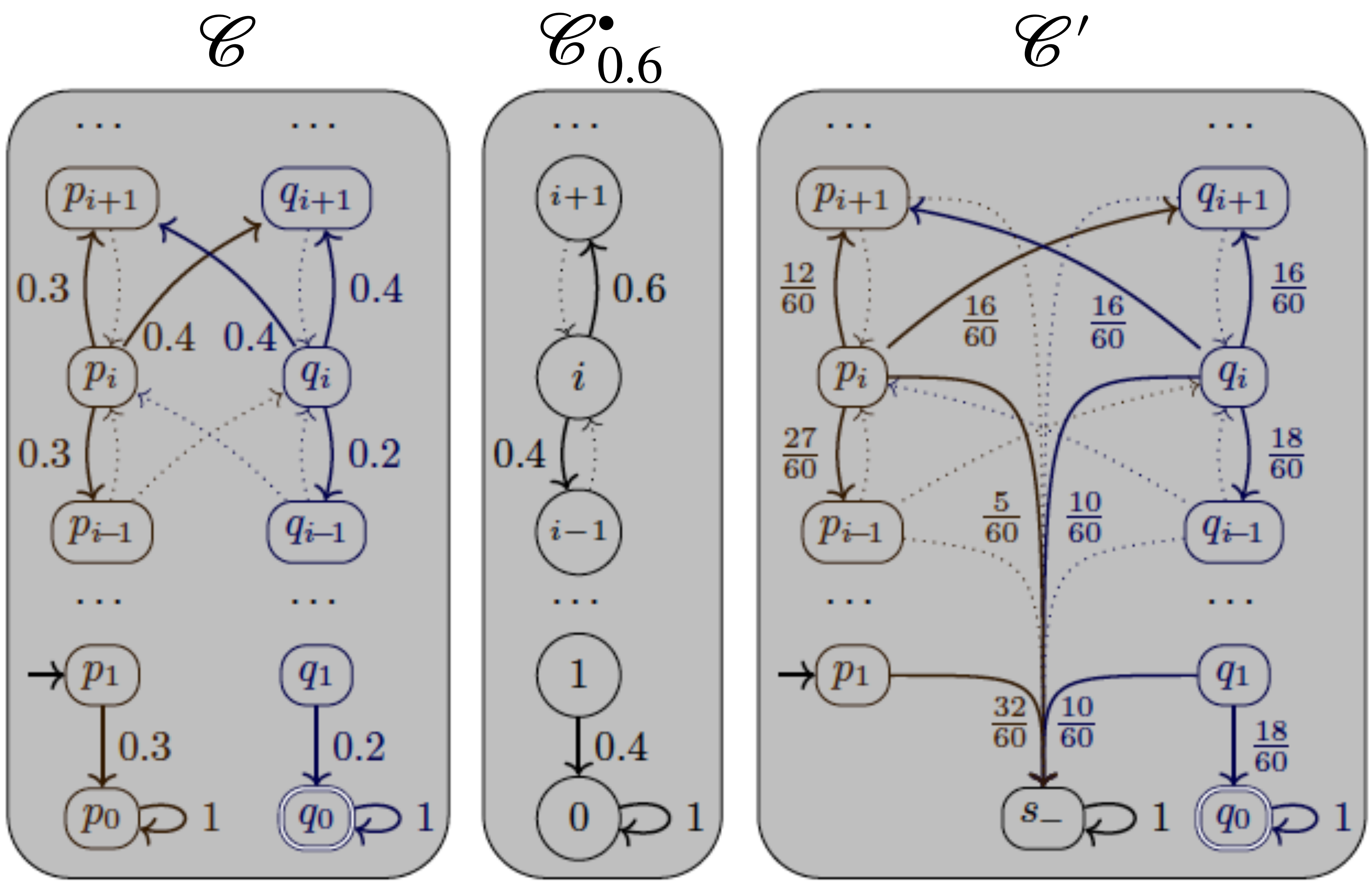

$\mathscr{C}$ is not decisive

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Application to probabilistic pushdown automata

[EKM06] J. Esparza, A. Kucera, R. Mayr. Model checking pushdown automatas (LMCS)

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- Can be seen as a layered Markov chain, using the length of the stack content

[EKM06] J. Esparza, A. Kucera, R. Mayr. Model checking pushdown automatas (LMCS)

Application to probabilistic pushdown automata

- Automaton with a stack
- Transition rules of the type $q \xrightarrow{?a!w} q'$ with $a \in \Sigma$ and $w \in \Sigma^{\leq 2}$
- Probabilities given by weights
  - In general: $W(t, w)$ with t a transition and $w \in \Sigma^*$ a stack content
  - [EKM06]: W only depends on t
  - Polynomial weight: $W(t, w)$ is a polynomial in $|w|$

$$A \xrightarrow{1} C \quad A \xrightarrow{n} BB \quad B \xrightarrow{5} \varepsilon$$
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Analysis can be done using the first-order theory of the reals

- Can be seen as a layered Markov chain, using the length of the stack content
- Local conditions on transition rules to ensure the hypotheses of the theorem

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Implementation

https://cosmos.lacl.fr/

[BBDHP15] P. Ballarini, B. Barbot, M. Duflot, S. Haddad, N. Pekergin. Hasl: A new approach for performance evaluation and model checking from concepts to experimentation (Performance Evaluation)

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- Some experiments have been done

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The larger is p, the larger is N_0

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  - Use the abstraction $\mathscr{C}_p^\bullet$ with desactivation zone $F^\bullet = [0; N_0]$

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Is there a best p?

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- p big: large desactivation zone (N_0)
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Note: in all experiments, the confidence is set to $99\ \%$

First example

- State-free proba. pushdown automaton $\mathscr{C}$
  $A \xrightarrow{1} C \quad A \xrightarrow{n} BB \quad B \xrightarrow{5} \varepsilon$
  $B \xrightarrow{n} AA \quad C \xrightarrow{1} C$
- Start from A, and target the empty stack

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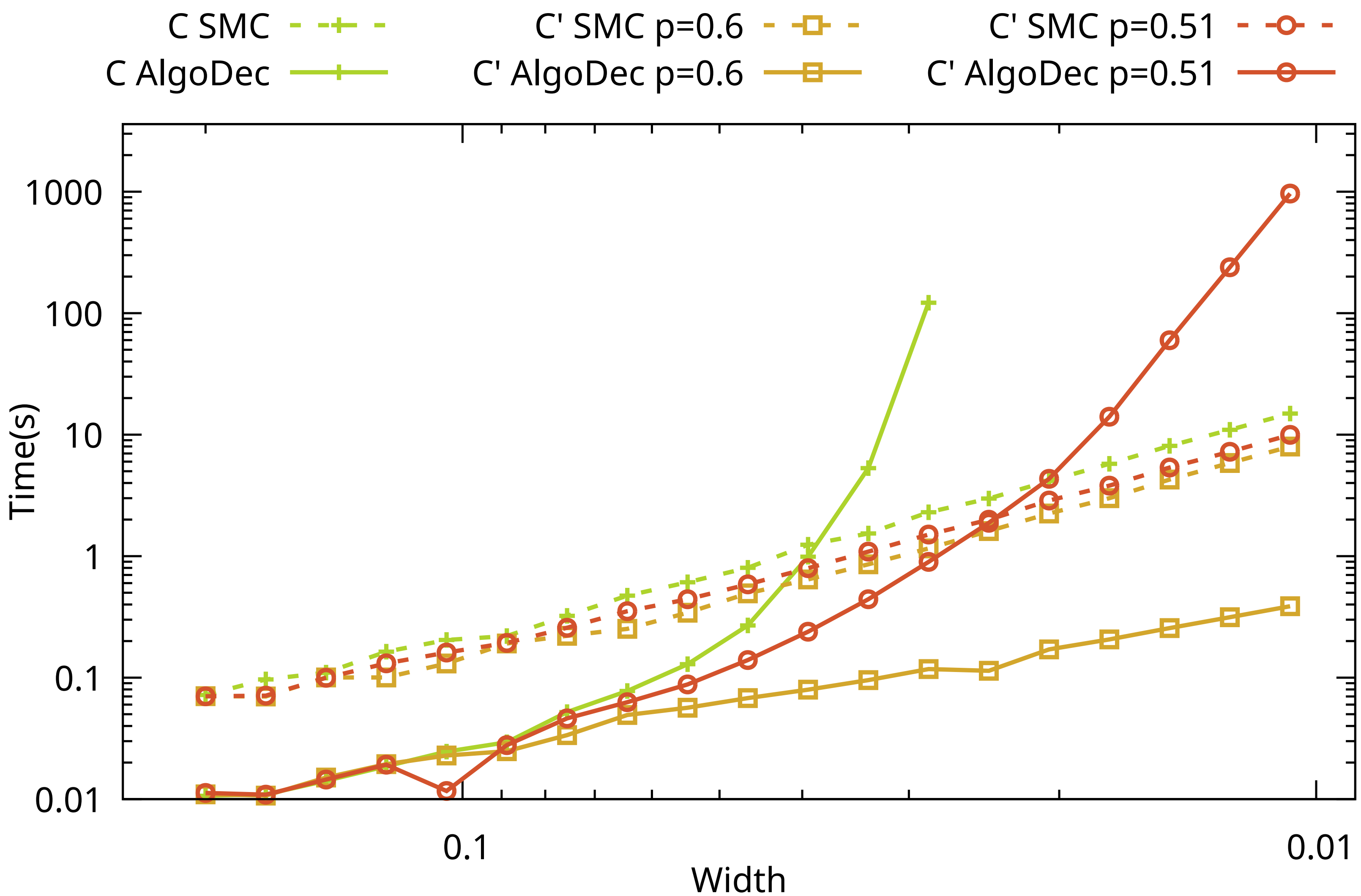
- It is decisive
- It is p-divergent for every $1/2 < p < 1$

First example
Experimental results



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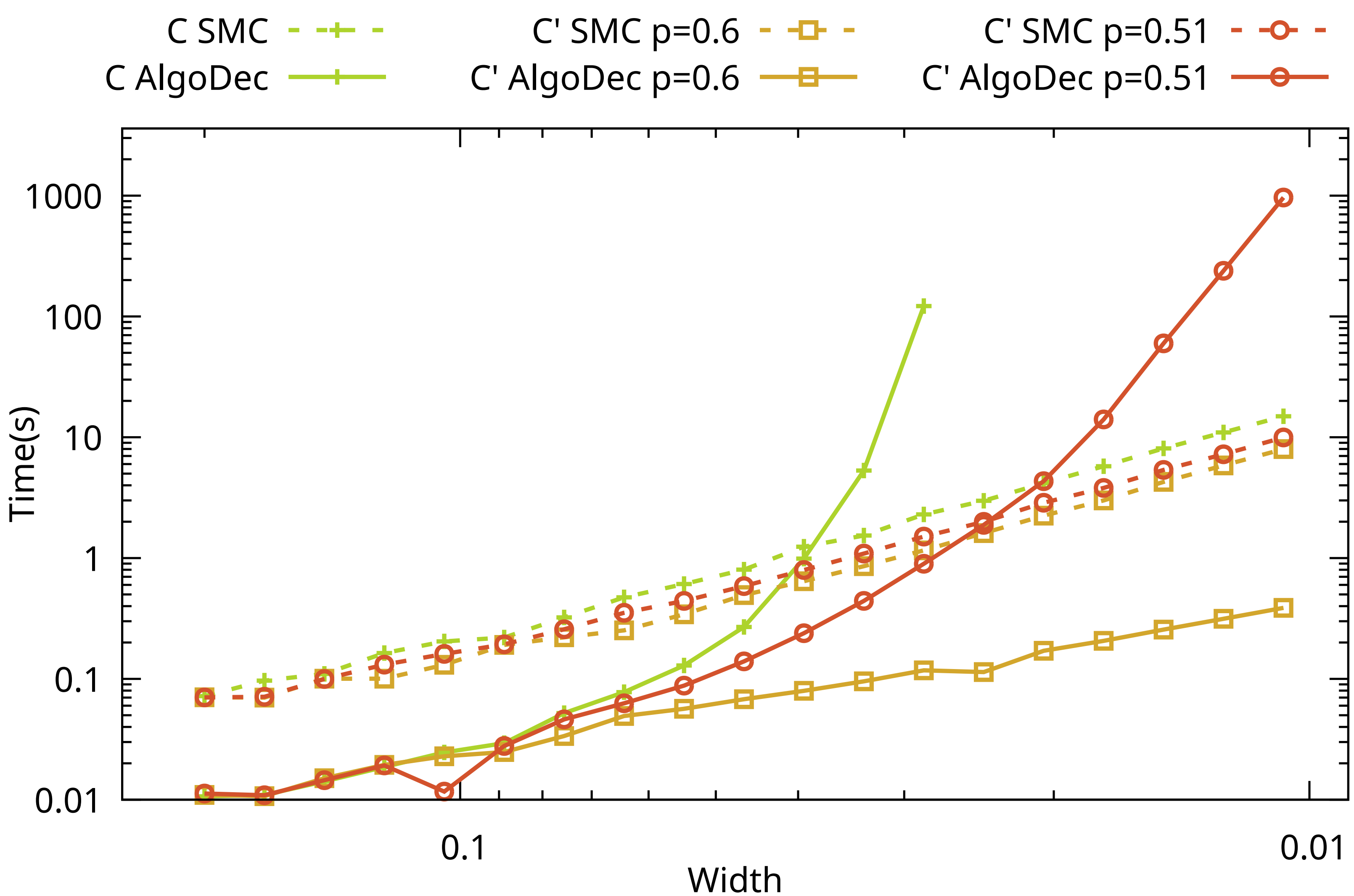

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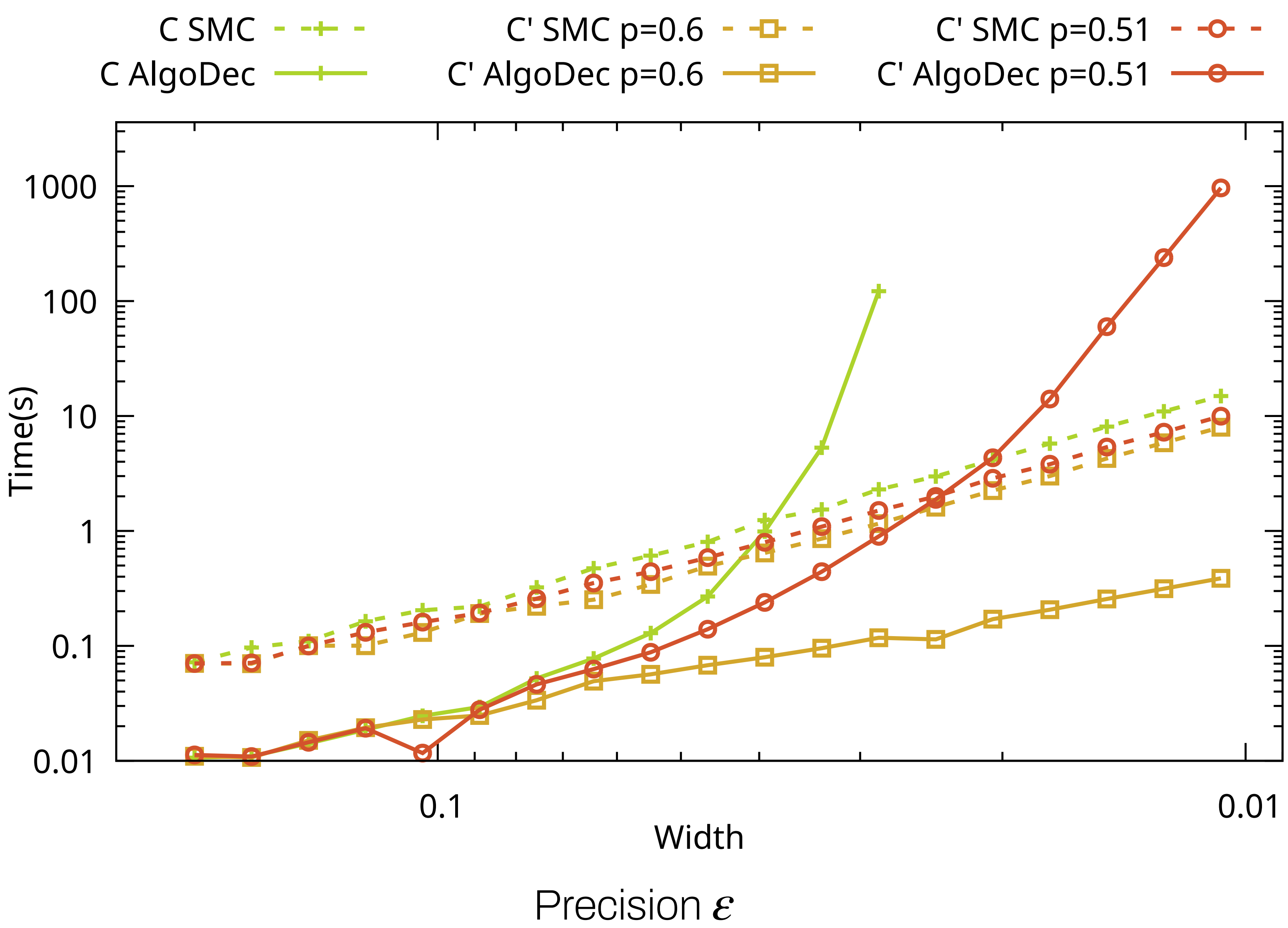

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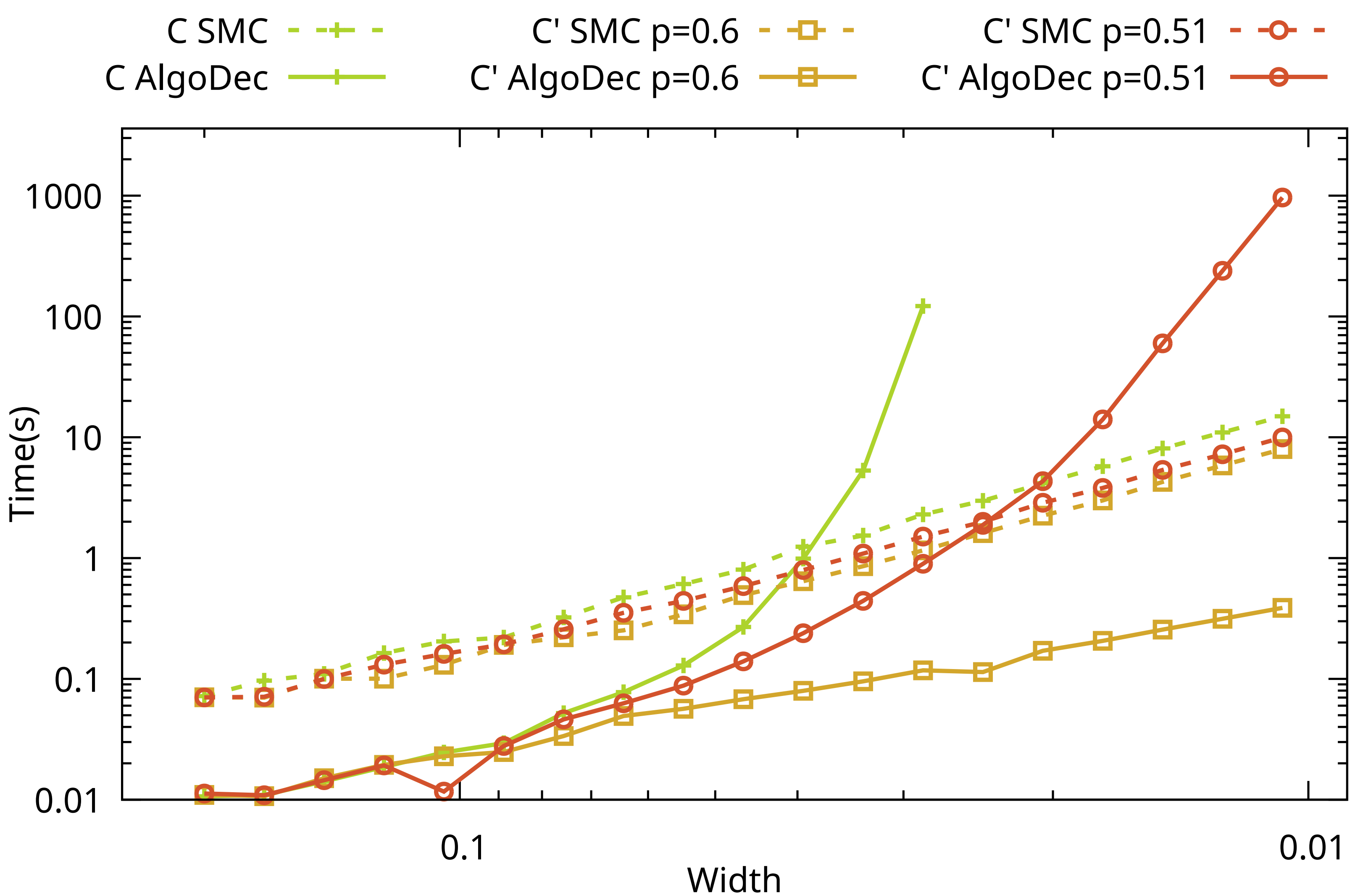

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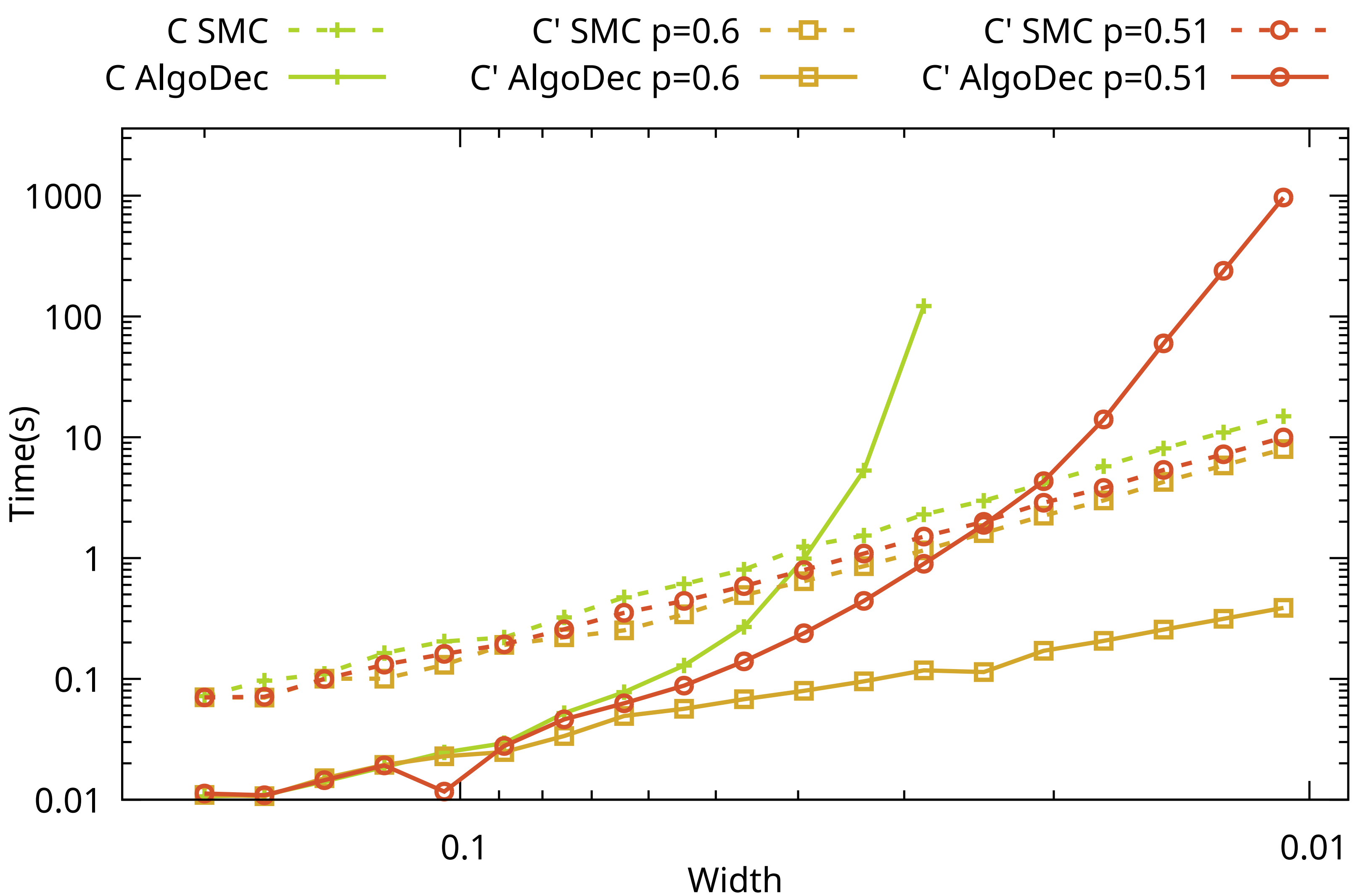

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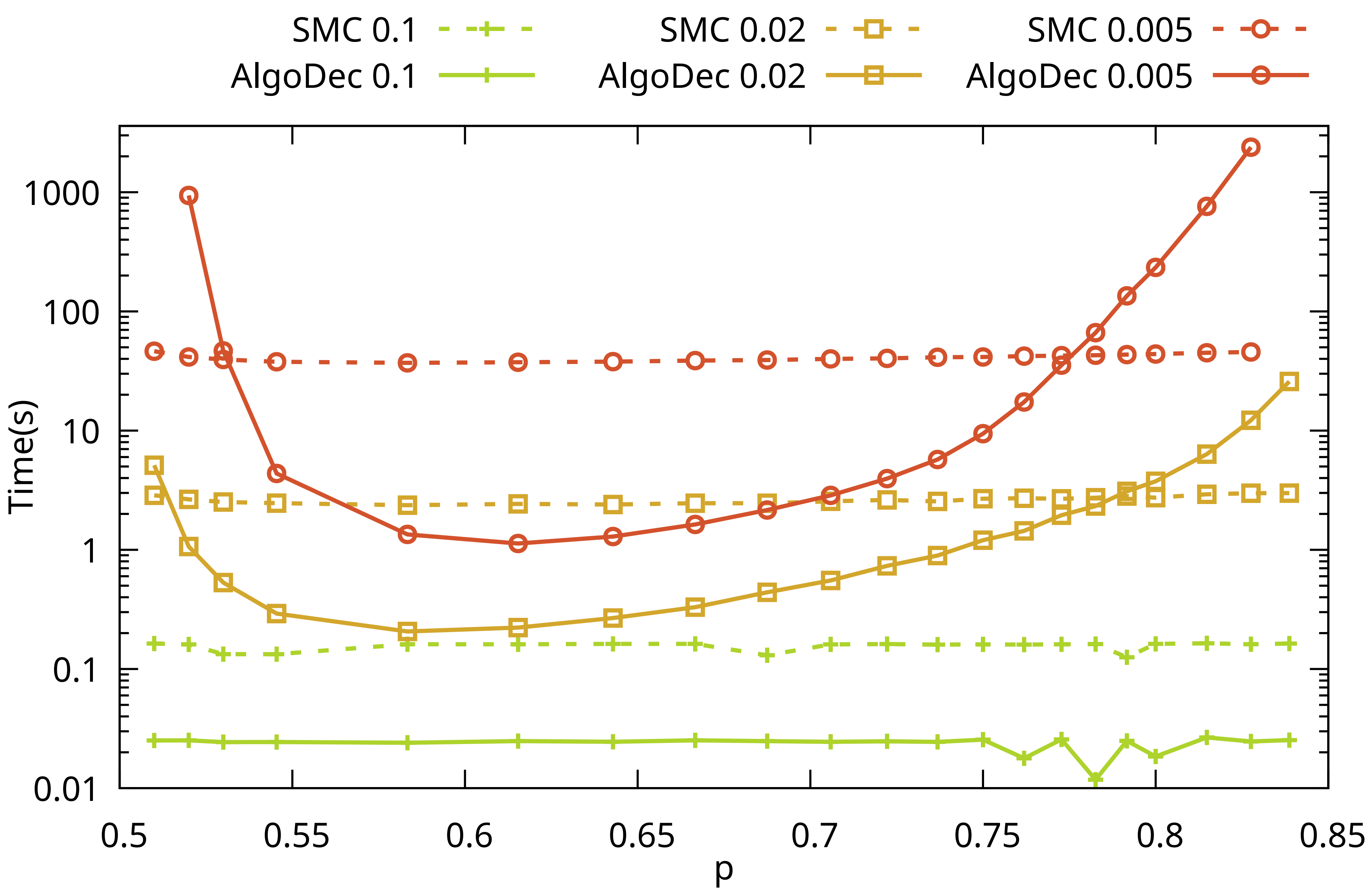
- There seems to be « a best p » ($p = 0.6$ here)
- For that best p, Approx behaves very well!

First example — continued



Second example

- State-free proba. pushdown automaton $\mathscr{C}$
  $A \xrightarrow{1} B \quad A \xrightarrow{1} C \quad B \xrightarrow{10} \varepsilon \quad B \xrightarrow{10+n} AA$
  $C \xrightarrow{10} A \quad C \xrightarrow{10+n} BB$

- Start from A, and target the empty stack

> - It is not decisive
> - It is p-divergent for every $1/2 < p < 1$

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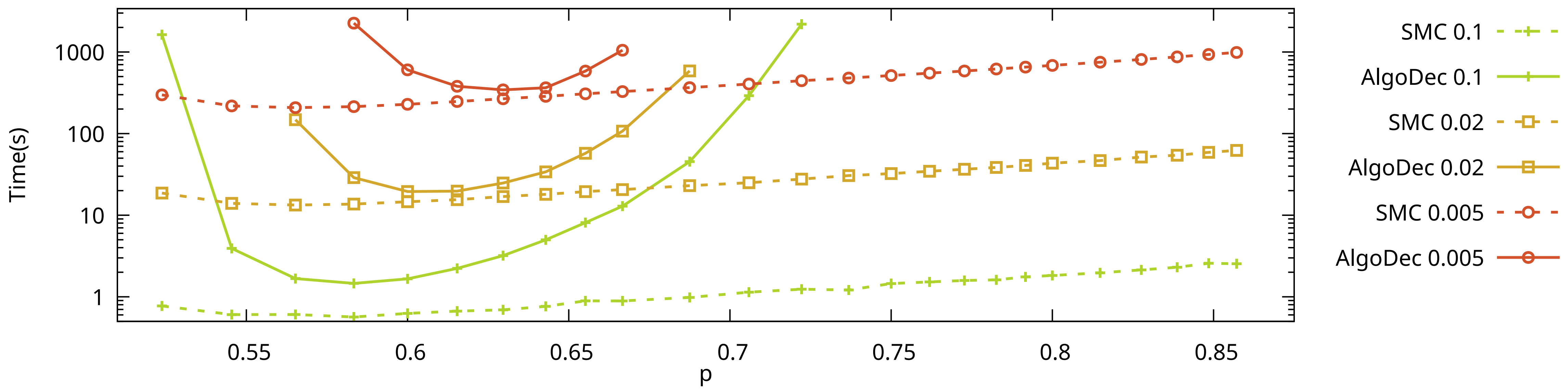

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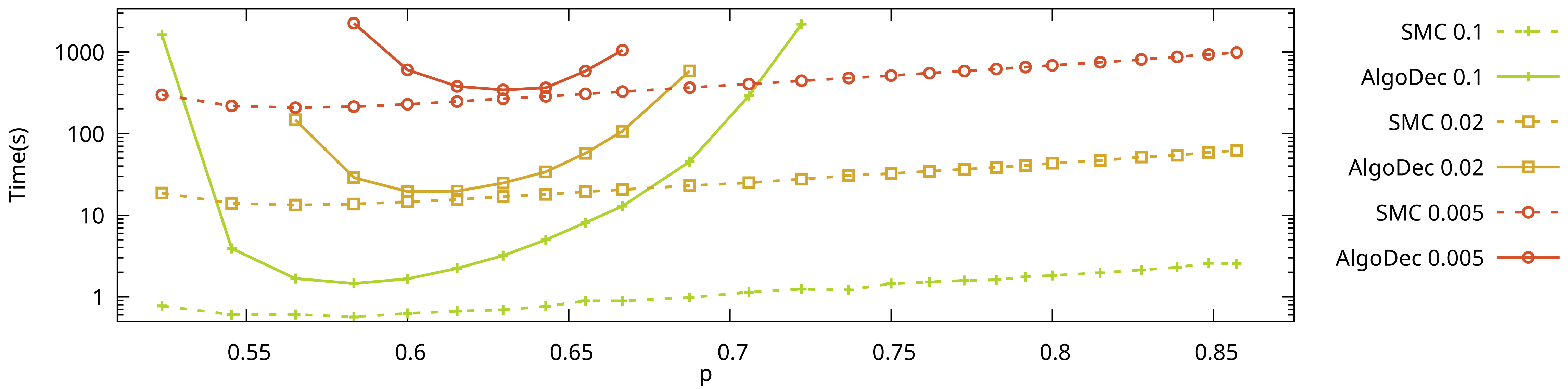

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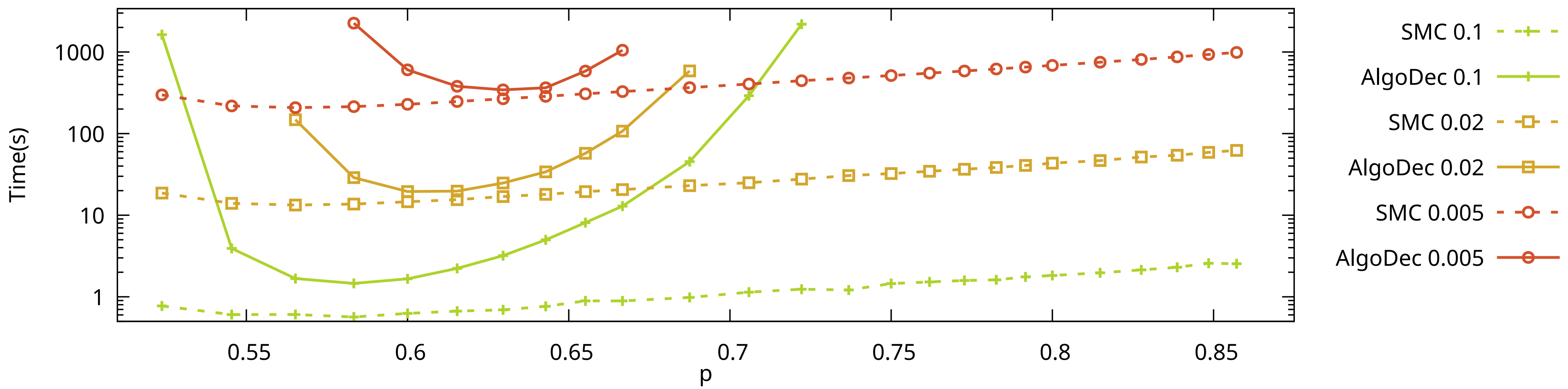

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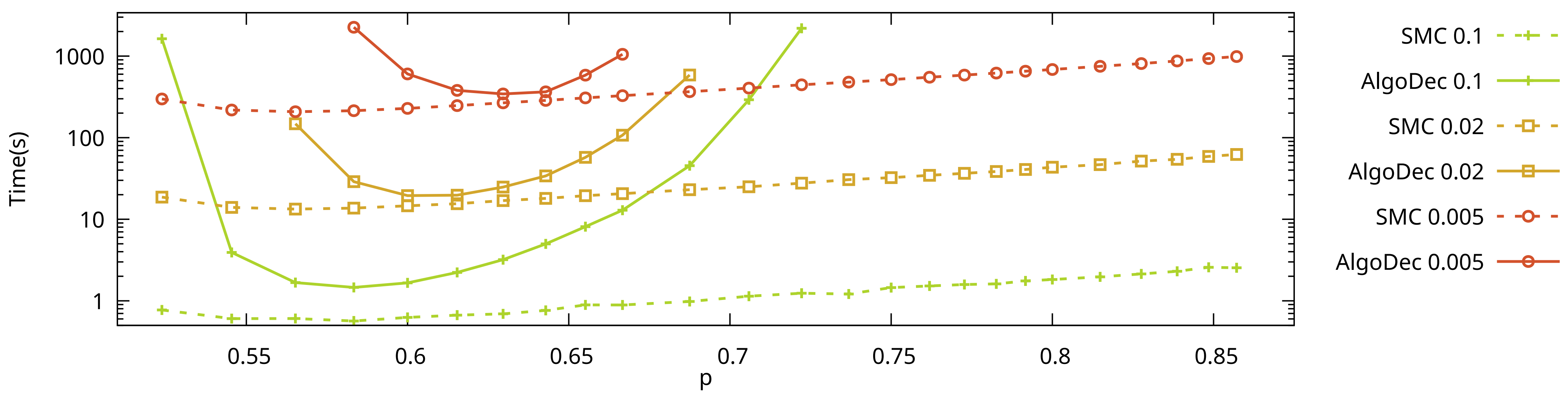

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- Approx very sensitive to p

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Deterministic guarantees

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Some smoother conditions for application of the approach?

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